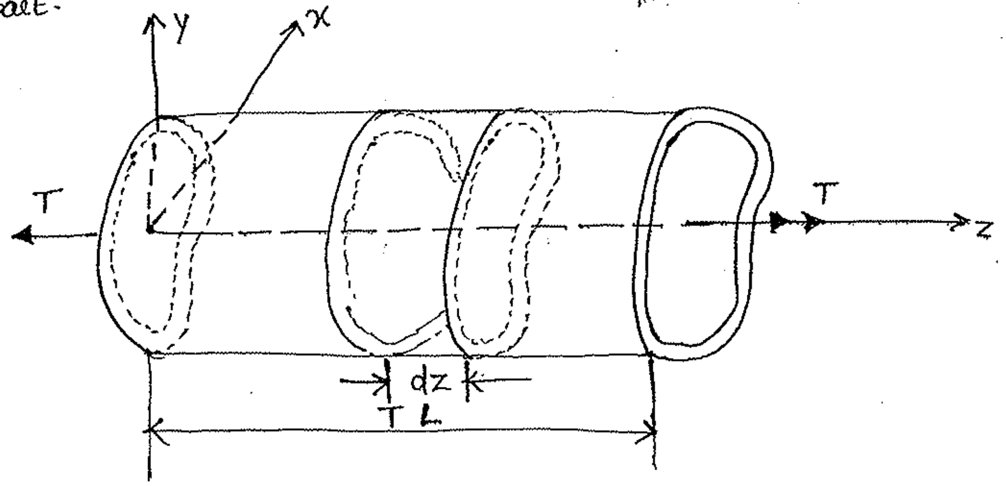
# Unit - III Shear flow in closed sections

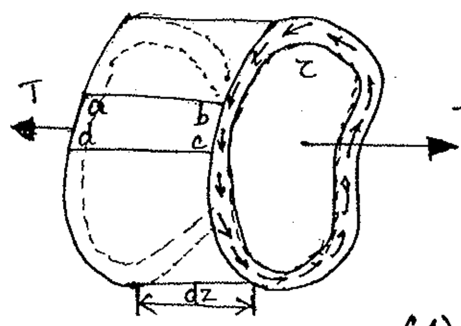
Bredt - Batho formula, Single and multi cell Structures. Approximate methods. Shear flow in single & multi cell Structures under torsion. Shear flow in Single and multicell under bending with walls effective and ineffective.

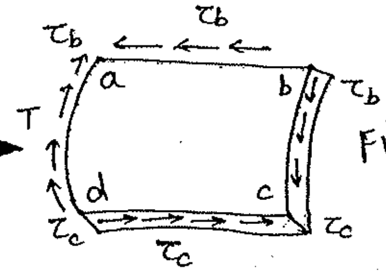
Introduction: - Thin walled tubes

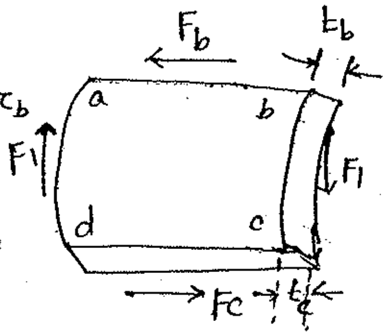
In light weight structures, Such as aircraft and spacecraft, thin walled tubular members of non-circular Shapes are often required to resist torsion.

To obtain formulae that are applicable to variety of shapes, let us consider a thin walled tube of arbitrary cross-Sectional shape. The tube is cylindrical (ie all cross Sections have Same dimensions) and is Subjected to pure torsion by torques T acting at the ends. The Thickness "t" of the wall may vary around the cross Section, but t is assumed to be Small in comparison with the total width of the Section. The Shear stress "$\tau$" acting on the cross section is pictured in the figure, which shows an element of the tube cut out between two cross sections at a distance dz apart.



The Shear stresses are directed parallel to the edges of the cross Section, and they flow around the tube. The intensity of the shear Stresses Varies so slightly across the thickness of the tube (because the tube is assumed to be thin) ie for many Purposes, we may assume $\tau$ to be constant across the thickness. However, the manner in which $\tau$ Various alound The cross Section must be determined from equilibrium considerations.

To determine the magnitude of the Shear stresses, consider a rectangular element Obtained by taking two longitudinal cuts ab and cd. This element is isolated as a free body in figure. Acting across the cross Sectional face bc are the Shear stress $\tau$ as Shown in figure. It is assumed that these stresses may Vary in intensity as we move along the cross Section from b to c. Thus at b the Shear stress is denoted as $\tau_b$ and at c it is denoted as $\tau_c$.

As we know from equilibrium, identical shear stresses acts in the opposite direction on the other cross sectional face "ad". On the longitudinal faces ab and cd, the will act, shear stresses of Same magnitude as those on The cross Sections, Same as shear Stresses on Perpendicular Planes are equal in magnitude. Thus, the consultant Shear stress on faces ab and cd are equal to $\tau_b$ and $\tau_c$ respectively.

The Shear stresses acting on the longitudinal faces produces force $F_b$ and $F_c$, that can be obtained by multiplying The Stresses by the areas on which They acts, thus $F_b = \tau_b t_b dz$ ; $F_c = \tau_c t_c dz$

in which $t_b$ and $t_c$ represents The thickness of the tube at b and c, respectively. In addition, forces $F_1$ are produced by the stresses acting on faces bc and ad, but the forces do not enter into our discussion. From the equilibrium of the element in z direction, we say That $F_b = F_c$

(or) $\tau_b t_b = \tau_c t_c$. Because the location of the longitudinal cuts ab and cd were Selected arbitrarily, it follows from The proceeding equation that the product of the Shear Stress $\tau$ and the thickness $t$ of the tube is the Same at every point in the cross section. This Product is known as the shear flow and is denoted by the letter $q$.

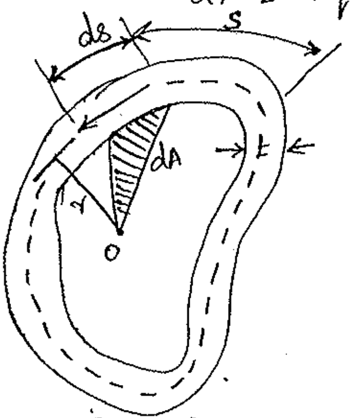
$$q = \tau_t = \text{constant}.$$

Thus, the largest shear stress occurs where the thickness of the tube is smallest and vice versa. Of course, in regions where thickness is constant, the shear stress is also constant.

The next step in the analysis is to relate the shear flow q (and hence the stress τ) to the torque T acting on the tube.

Consider an element of area of length ds in the cross section. The distance s is measured along the median line of the cross section (shown as a dash lines in the fig). The total shear force acting on the element of area q ds. and the moment of this force about any point O is

$$dT = rq\,ds.$$




Cross Section of a thin walled tube.

$$dT = \text{Elemental force} \times \text{radius}$$
$$= dF \times r.$$
$$= \tau dA \times r$$
$$= \tau ds \times t \times r$$
$$= \tau t\, ds \times r$$
$$= q\,ds \times r$$
$$\boxed{dT = rq\,ds}$$

in which r is perpendicular distance from O to the line of action of the force. The later is the tangent to the median line of the cross section at the element ds.

The total torque T produced by the shear stresses, is obtained by integrating the full length "L" of the median line of the cross section

$$\int dT = q\int_0^L r\,ds$$

$$\boxed{T = q\int_0^L r\,ds}$$

The integral in the expression has a simple geometric interpretation. The quantity rds represents twice the area of the shaded triangle shown in figure. Note that the triangle has a base length ds and a height equal to r. Therefore, the integral represents double the Area A enclosed by the median line of the cross section, thus

$$T = q(2A) \Rightarrow T = 2Aq.$$

(or) $dA = \frac{1}{2} r ds \Rightarrow r ds = 2dA.$

$$\int dT = \int q r ds = \int q (2dA) \Rightarrow T = q2A$$

$$\boxed{T = 2qA}$$

From the equation we get

$$\boxed{q = \tau t = \frac{T}{2A}}$$

$$\boxed{\tau = \frac{q}{t} = \frac{T}{2tA}}$$

This equation is known as Bredt - Batho formula.

From these equations, the shear flow $q$ and the shear stress $\tau$ can be calculated for any thin walled tube.

The angle of twist $\phi$ can be calculated by considered the strain energy of the tube. Because elements of the tube are in pure shear, the strain energy is $\tau^2/2G$. Hence the strain energy of a small element of the tube having cross - sectional area tds and length dz is

$$dU = \frac{\tau^2}{2G} \times \text{Volume}$$

$$= \frac{\tau^2}{2G} t ds\, dz$$

Multiply and divide by $t$

$$\therefore dU = \frac{\tau^2 t^2}{2G} \frac{ds}{t} dz$$

$$dU = \frac{q^2}{2G}\frac{ds}{t}dz \quad [\because q = \tau t]$$

$\therefore$ the total strain energy,

$$U = \int dU = \frac{q^2}{2G}\int_0^L \left[\int_0^L dz\right]\frac{ds}{t}$$

in which we have utilized the fact that the shear flow $q$ is a constant and may be placed outside the integral sign. Also, we note that $t$ may vary with position around the median line, hence it must remain under the integral sign with $ds$. The inner integral is equal to the length $L$ of the tube, hence the equilibrium for $U$ becomes,

$$U = \frac{q^2 L}{2G}\oint_0^L \frac{ds}{t} \quad \left[\because \int_0^L dz = L\right]$$

By substituting $q = \frac{T}{2A}$

$$U = \frac{T^2 L}{8A^2 G}\oint_0^L \frac{ds}{t}$$

Castigliano's Theorem:

Theorem - I $\quad P_i = \frac{\partial U}{\partial \delta_i}$

The equation is called castigliano's first theorem. It states that the partial derivative of the strain energy with respect to any displacement $\delta_i$ is equal to the corresponding force $P_i$, provided that the strain energy is expressed as a function of displacement.

Theorem - II $\quad \delta_i = \frac{\partial U}{\partial P_i}$

This equation is said to be castigliano's II theorem. It states that for a linear structure the partial derivative of the strain energy with respect to any load $P_i$ is equal to the corresponding displacement $\delta_i$, By castigliano's Second theorem, the angle of twist $\phi$ can be calculated by the partial derivative of the strain energy with respect to torque T. Provided that the strain energy is expressed as a function of the loads.

$$\therefore \phi = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}\left[\frac{T^2 L}{8A^2 G}\int_0^L \frac{ds}{t}\right]$$

$$= \frac{2TL}{8A^2 G}\oint_0^L \frac{ds}{t}$$

$$\boxed{\phi = \frac{TL}{4A^2 G}\oint_0^L \frac{ds}{t}} \rightarrow (1)$$

$\therefore$ The angle of twist per unit length $(\beta)$ can be calculated as

$$\beta = \frac{\phi}{L} = \frac{T}{4A^2 G}\oint_0^L \frac{ds}{t}$$

$$\beta = \frac{T}{2A} \times \frac{1}{2AG}\oint_0^L \frac{ds}{t}$$

$$\boxed{\beta = \frac{q}{2AG}\oint_0^L \frac{ds}{t}}$$

where $G = \frac{E.m}{2(m+1)}$

$\frac{1}{m}$ = Poisson's ratio

A = Area.

E = young's modulus.

For a solid shaft,

$$\frac{G\theta}{l} = \frac{T}{J} \Rightarrow \theta = \frac{TL}{GJ} \rightarrow (b)$$

Comparing the equation (a) with (b)

$$\frac{1}{J} = \frac{1}{4A^2}\int_0^L \frac{ds}{t}$$

$$\boxed{J = \frac{4A^2}{\int_0^L \frac{ds}{t}}}$$

## Shear flow due to Torque

Problem - 1.

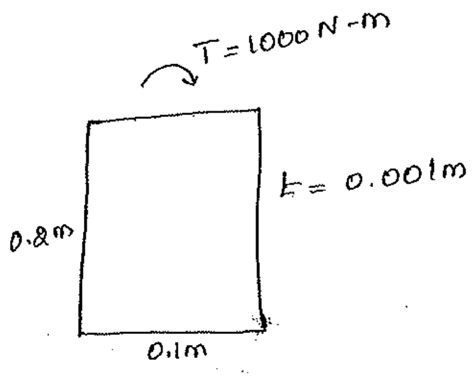
Calculate the Shear flow, Shear stress and twist per unit length for the given section. Assume $G = 25 \times 10^9 N/m^2$ and $T = 1000 Nm$

Data given

$G = 25 \times 10^9 N/m^2$

$T = 1000 \, N-m$

$t = 0.001 \, m$




Soln

Area $= 0.1 \times 0.2 = 0.02 \, m^2$

$T = 2Aq = 2 \times 0.02 \times q$

$\therefore \boxed{q = 25000 \, N/m}$

WKT $q = \tau \times t$

$\therefore \tau = \frac{q}{t} = \frac{25000}{0.001}$

Shear stress, $\boxed{\tau = 25 \times 10^6 N/m^2}$

Twist per unit length, $\beta = \frac{1}{2AG} \oint \frac{q}{t} ds.$

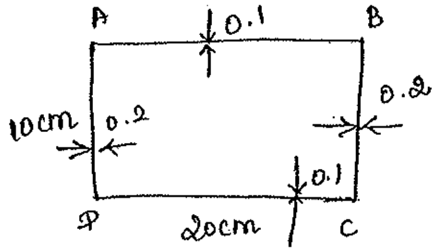
$$\beta = \frac{1}{2 \times 0.02 \times 25 \times 10^9} \oint \frac{25000}{0.001} ds$$

$$= \frac{\cancel{25} \times 10^6}{2 \times 0.02 \times \cancel{25} \times 10^{\cancel{9} 3}} [S]$$

$$= \frac{1}{0.04 \times 10^3} [0.1 + 0.2 + 0.1 + 0.2]$$

$\boxed{\beta = 0.015 \text{ rad/m}}$

Problem: 2 Calculate the shear flow, shear stress, twist per unit length. Assume $G = 25 \times 10^9 N/m^2$. $T = 100$ N.m. Section of Varying thickness is shown below.




Area $A = 20 \times 10 = 200\,cm^2 = 0.02\,m^2$

$$q = \frac{T}{2A} = \frac{100}{2 \times 0.02}$$

$$q = 2500\,N/m$$

$$q = \tau \times t \Rightarrow \tau = q/t$$

$$\tau_{AB} = \frac{q}{t_{AB}} = 2.5 \times 10^6\,N/m^2$$

$$\tau_{BC} = \frac{q}{t_{BC}} = 1.25 \times 10^6\,N/m^2$$

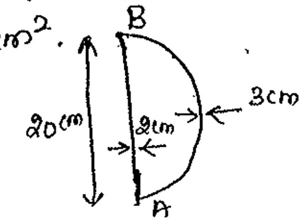
$$\beta = \frac{1}{2AG} \oint \frac{q}{t}\,ds.$$

$$= \frac{q}{2AG}\left[\frac{S_{AB}}{t_{AB}} + \frac{S_{BC}}{t_{BC}} + \frac{S_{CD}}{t_{CD}} + \frac{S_{DA}}{t_{DA}}\right]$$

$$= \frac{2500}{2 \times 0.02 \times 25 \times 10^9}\left[\frac{0.2}{0.001} + \frac{0.1}{0.002} + \frac{0.2}{0.001} + \frac{0.1}{0.002}\right]$$

$$\boxed{\beta = 1.25 \times 10^{-3}\,rad/m}$$

Problem: 3 Calculate the shear flow and shear stress, shear twist per unit length of the given section. $T = 6000\,N\text{-}cm$, $G = 30 \times 10^5\,N/cm^2$.



$T = 2Aq$

$q = \dfrac{T}{2A}$

$A = \dfrac{\pi r^2}{2} = \dfrac{\pi \times 10^2}{2}$

$= 157.07 \text{ cm}^2$

$$q = \frac{6000}{2 \times 157.07} = 19.09 \text{ N/cm}$$

$q = \tau t$

$\tau = q/t$

Shear stress across AB, $\tau_{AB} = \dfrac{q}{t_{AB}} = \dfrac{19.09}{0.2} = 95.492 \text{ N/cm}^2$

Shear stress across BA, $\tau_{BA} = \dfrac{q}{t_{BA}} = \dfrac{19.09}{0.3} = 63.63 \text{ N/cm}^2$

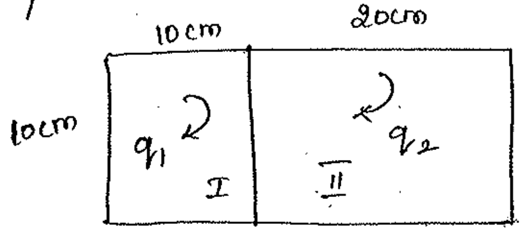
Twist per unit length, $\beta = \dfrac{1}{2AG} \times q \left[\dfrac{AB}{t} + \dfrac{BA}{t}\right]$

$$\beta = \frac{1 \times 19.09}{2 \times 157.07 \times 30 \times 10^5} \left[\frac{20}{0.2} + \frac{\pi r}{0.3}\right]$$

$$= 2.0256 \times 10^{-8} \left[\frac{20}{0.2} + \frac{\pi \times 10}{0.3}\right]$$

$$\boxed{\beta = 4.146 \times 10^{-6} \text{ rad/cm}}$$

Problem: 4 Calculate the shear flow for the given section and calculate $\beta$.




$T = 1000$ N-cm

$t = 2$ mm

$G = 25 \times 10^9$ Pa.

$A_1 = 0.1 \times 0.2 = 0.01 m^2$ | $A_2 = 0.1 \times 0.2 = 0.02 m^2$

$T = 2A_1 q_1 + 2A_2 q_2$

$10 = 0.02 q_1 + 0.04 q_2 \longrightarrow (1)$

Now for first cell,

$$\beta_1 = \frac{1}{2A_1 G t}\left[0.1 \times q_1 + 0.1 q_1 + 0.1 q_1 + 0.1(q_1 - q_2)\right]$$

$$\beta_1 = \frac{1}{2 \times 0.01 \times 25 \times 10^9 \times 2 \times 10^{-3}}\left[0.4 q_1 - 0.1 q_2\right] \longrightarrow (2)$$

Now for second cell,

$$\beta_2 = \frac{1}{2A_2 G t}\left[q_2 \times 0.2 + q_2 \times 0.1 + q_2 \times 0.2 + 0.1(q_2 - q_1)\right]$$

$$= \frac{1}{2 \times 0.02 \times 25 \times 10^9 \times 2 \times 10^3}\left[0.6 q_2 - 0.1 q_1\right]$$

$$\beta_2 = 0.5 \times 10^{-6}\left[0.6 q_2 - 0.1 q_1\right] \longrightarrow (3)$$

Compatibility condition $\beta_1 = \beta_2$

$$1 \times 10^6\left[0.4 q_1 - 0.1 q_2\right] = 0.5 \times 10^{-6}\left[0.6 q_2 - 0.1 q_1\right]$$

$$0.8 q_1 - 0.2 q_2 = 0.6 q_2 - 0.1 q_1$$

$$0.9 q_1 - 0.8 q_2 = 0 \longrightarrow (4)$$

Solving eqn (1) and (4) $\boxed{q_1 = 153.846 \text{ N/m} \quad q_2 = 173.076 \text{ N/m}}$

from (2) & (3)

$$\beta_1 = 1 \times 10^{-6}\left[(0.4 \times 153.846) - (0.1 \times 173.076)\right]$$

$$\boxed{\beta_1 = 4.423 \times 10^5 \text{ rad/m}}$$

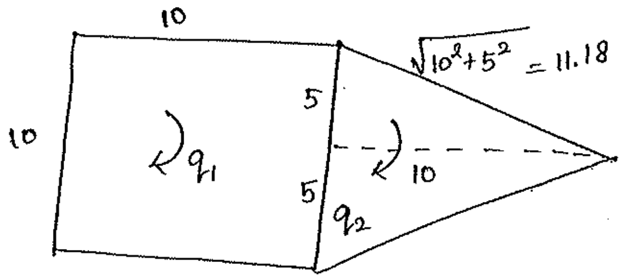
To check

$$\beta_2 = 0.5 \times 10^6\left[(0.6 \times 173.076) - (0.1 \times 153.846)\right]$$

$$\boxed{\beta_2 = 4.423 \times 10^5 \text{ rad/m}}$$

[10]

Problem: 5 Calculate the Shear flow for the given Section.




All dimensions are in cm.

$T = 10000$ N-cm

$A_1 = 10 \times 10 = 100 \text{ cm}^2$

$A_2 = 2\left(\frac{1}{2} \times 10 \times 5\right) = 50 \text{ cm}^2$

$T = 2A_1 q_1 + 2A_2 q_2$

$10000 = 200 q_1 + 100 q_2 \rightarrow (1)$

For cell I ; $\beta_1 = \frac{1}{2A_1 Gt}\left[10q_1 + 10q_1 + 10q_1 + 10(q_1 - q_2)\right]$

$\beta_1 = \frac{1}{2A_1 Gt}\left[40q_1 - 10q_2\right] \rightarrow (2)$

For cell - II ; $\beta_2 = \frac{1}{2A_2 Gt}\left[11.18 q_2 + 11.18 q_2 + 10(q_2 - q_1)\right]$.

$\beta_2 = \frac{1}{2A_2 Gt}\left[32.36 q_2 - 10q_1\right] \rightarrow (3)$

For compatability condition

$\beta_1 = \beta_2$

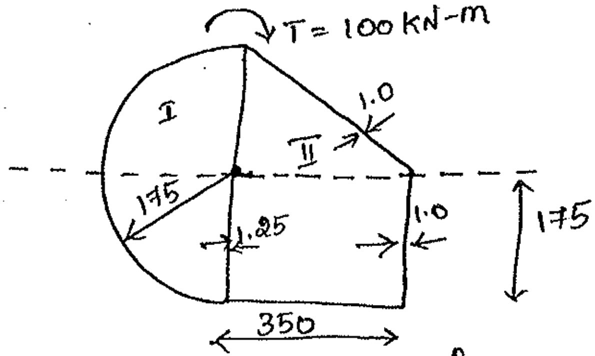
$$\frac{1}{2A_2 Gt}(40q_1 - 10q_2) = \frac{1}{2A_2 Gt}(32.36 q_2 - 10q_1)$$

$$\frac{1}{100}(40q_1 - 10q_2) = \frac{1}{50}(32.36 q_2 - 10q_1) \rightarrow (4)$$

Solving (1) & (4) we get.

$$\boxed{\begin{aligned} q_1 &= 35.67 \text{ N/cm} \\ q_2 &= 28.65 \text{ N/cm} \end{aligned}}$$

Problem : 6 Find the shear flow and $\beta$ for the given figure




All dimensions are in mm.

Data given :-

$G = 25 \times 10^9 N/m^2$

$A_1 = \frac{\pi \times 175^2}{2} = 0.048 m^2$

$A_2 = (0.175 \times 0.350) + (\frac{1}{2} \times 0.350 \times 0.175)$

$A_2 = 0.0918 m^2$

$T = 2A_1 q_1 + 2A_2 q_2$

$100000 = 0.096 q_1 + 0.1836 q_2 \rightarrow (1)$

For First cell,

$$\beta_1 = \frac{1}{2A_1 G}\left[\frac{\pi \times 0.175}{0.001} \times q_1 + \frac{0.35 (q_1 - q_2)}{1.25 \times 10^{-3}}\right]$$

$$= \frac{1}{2A_1 G}\left[549.778 q_1 + 280 q_1 - 280 q_2\right]$$

$$\beta_1 = \frac{1}{2A_1 G}\left[829.778 q_1 - 280 q_2\right] \rightarrow (2)$$

At cell II,

$$\beta_2 = \frac{1}{2A_2 G}\left[\frac{0.35 q_2}{0.001} + \frac{0.175 q_2}{0.001} + \frac{0.3913 q_2}{0.001} + \frac{0.35 (q_2 - q_1)}{0.00125}\right]$$

$$\beta_2 = \frac{1}{2A_2 G}\left[1196.3 q_2 - 280 q_1\right] \rightarrow (3)$$

For Compatability condition:

$$\beta_1 = \beta_2$$

$$\frac{1}{2A_1 G}(829.778 q_1 - 280 q_2) = \frac{1}{2A_2 G}(1196.3 q_2 - 280 q_1)$$

$$829.778q_1 - 280q_2 = \frac{0.048}{0.0918}(1196.3q_2 - 280q_1)$$

$$829.778q_1 - 280q_2 = 625.5q_2 - 146.4q_1$$

$$976.178q_1 - 905.5q_2 = 0 \rightarrow (4)$$

Solving (1) and (4) we get

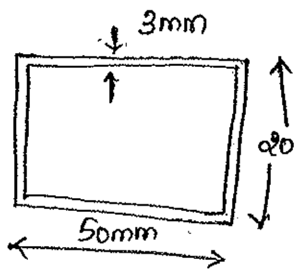
$$\boxed{\begin{aligned} q_1 &= 3.4021 \times 10^5 \text{ N/m} \\ q_2 &= 3.66 \times 10^5 \text{ N/m} \end{aligned}}$$

$$\beta_1 = \beta_2 = \frac{1}{2 \times 0.0918 \times 25 \times 10^9}\left[1196.3 \times 3.66 \times 10^5 - 280 \times 3.4 \times 10^5\right]$$

$$\boxed{\therefore \beta = 7.46 \times 10^{-2} \text{ rad/m}}$$

Problem: 7.

A thin walled hollow tube of rectangular cross-section is shown in figure. Calculate the maximum shear stress $\tau_{max}$ due to torque $T = 120$ N-m




Shear stress $\tau = \frac{q}{t} = \frac{T}{2tA}$

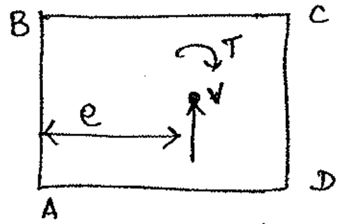
$A = 20 \times 50 = 1000$ mm

$$\therefore \tau = \frac{120 \times 1000}{2 \times 3 \times 1000} = 20 \text{ N/mm}^2.$$

Bending and torsion of a single cell

Let us consider a single closed cell beam subjected to lateral load V as shown in figure.

Let the shear centre be at the distance "e" from the web AB and hence the beam is Subjected to Shear load (V) passing through the Shear centre and torque of $T = Ve$




Let $q$ and $q_t$ be the Shear flow due to bending and torque

then $q_{net} = q + q_t$

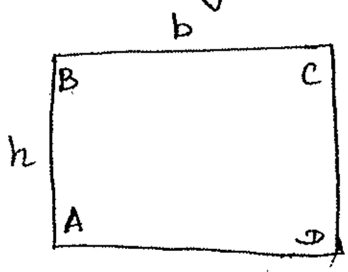
where $q = q_0 - \frac{V}{I}\int y t\, ds$ and

$$q_t = \frac{T}{2A}$$

where $q_0$ is the indeterminate Shear flow existing in the closed tube.

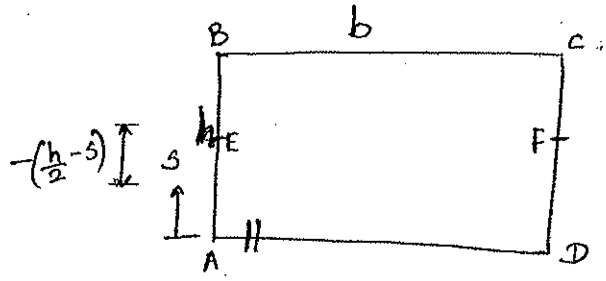
Problem: 8

Find the Shear flow of the closed Section shown in figure Subjected to a Vertical force V, passing through the Shear centre.




For closed Section, $q = q_0 - \frac{V}{I}\int y t\, ds$.

cut and open the tube along the generator at Some point to make it determinate.




$$q_{AB} = 0 - \frac{V}{I}\int_0^s -\left(\frac{h}{2}-s\right)t\,ds$$

$$= \frac{Vt}{2I}\left[hs - s^2\right].$$

@ Point A ; $s=0$ ; $q_A = 0$

@ point B ; $s=h$ ; $q_B = 0.$

@ Point E ; $s=\frac{h}{2}$ ; $q_E = \frac{Vth^2}{8I}$

$$q_{BC} = q_B - \frac{V}{I}\int\left(\frac{h}{2}\right)t\,ds = -\frac{Vhts}{2I} \quad (\because q_B = 0)$$

@ point B ; $s=0$ ; $q_B = 0$

@ point C ; $s=b$ ; $q_c = -\frac{Vhtb}{2I}$

$$q_{CD} = q_c - \frac{V}{I}\int_0^s\left(\frac{h}{2}-s\right)t\,ds = q_c - \frac{Vt}{I}\left[\frac{hs}{2} - \frac{s^2}{2}\right]$$

$$= -\frac{Vhtb}{2I} - \frac{Vt}{2I}\left(hs - s^2\right)$$

@ C ; $s=0$ ; $q_c = -Vhtb/2I$

@ D ; $s=h$ ; $q_D = -Vhtb/2I$

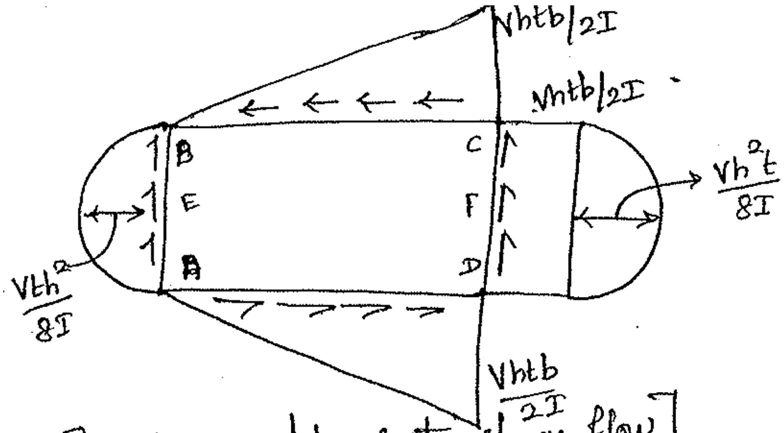
@ F ; $s=h/2$ ; $q_F = -\frac{Vhtb}{2I} - \frac{Vt}{2I}\left[\frac{h^2}{2} - \frac{h^2}{4}\right]$

$$= -\frac{Vhtb}{2I} - \frac{Vh^2t}{8I}$$

$$q_{DA} = q_D - \frac{V}{I}\int_0^s\left(\frac{-h}{2}\right)t\,ds = -\frac{Vhtb}{2I} + \frac{Vhts}{2I}$$

@ D ; $s=0$ ; $q_D = -Vhtb/2I$

@ A ; $s=b$ ; $q_A = 0$




[Statically determinate shear flow]

Let $q_0$ be the indeterminate shear existing at the cut before cutting

∴ For uncut section,

$$q_{AB} = q_0 + \frac{Vt}{2I}\left[hs - s^2\right]$$

$$q_{BC} = q_0 - \frac{Vhts}{2I}$$

$$q_{CD} = q_0 - \frac{Vhtb}{2I} - \frac{Vt}{2I}\left(hs - s^2\right)$$

$$q_{DA} = q_0 - \frac{Vhtb}{2I} + \frac{Vhts}{2I}$$

Twist per unit length ; $\beta = 0$ because the load passes through the shear flow

$$\beta = \frac{q}{2AG}\oint \frac{ds}{t} = \frac{1}{2AGt}\oint q\,ds$$

$$\beta = \frac{1}{2AGt}\left[\int_0^h q_{AB} + \int_0^b q_{BC} + \int_0^h q_{CD} + \int_0^b q_{DA}\right] ds.$$

$$= \frac{1}{2AGt}\left[\int_0^h \left[q_0 + \frac{Vt}{2I}\left(hs - s^2\right)\right] + \int_0^b \left[q_0 - \frac{Vhts}{2I}\right]\right.$$

$$\left. + \int_0^h \left[q_0 - \frac{Vhtb}{2I} - \frac{Vt}{2I}\left(hs - s^2\right)\right] + \int_0^b \left[q_0 - \frac{Vhtb}{2I} + \frac{Vhts}{2I}\right]\right] ds$$

$$= \frac{1}{2AGt}\left[\left\{q_0 s + \frac{Vt}{2I}\left(\frac{hs^2}{2} - \frac{s^3}{3}\right)\right\}_0^h + \left\{q_0 s - \frac{Vhts^2}{4I}\right\}_0^b\right.$$
$$+ \left\{q_0 s - \frac{Vhtbs}{2I} - \frac{Vt}{2I}\left(\frac{hs^2}{2} - \frac{s^3}{3}\right)\right\}_0^h + \left\{q_0 s - \frac{Vhtb}{2I}s + \left.\frac{Vhts^2}{4I}\right\}_0^b\right]$$

$$= \frac{1}{2AGt}\left[q_0(h+b+h+b) + \frac{Vt}{2I}\left(\cancel{\frac{h^3}{2}} - \cancel{\frac{h^3}{2}}\right)\right.$$
$$- \cancel{\frac{Vhtb^3}{4I}} - \frac{Vh^2tb}{2I} - \frac{Vt}{2I}\left(\cancel{\frac{h^3}{2}} - \cancel{\frac{h^3}{3}}\right)$$
$$\left. - \frac{Vhtb^2}{2I} + \cancel{\frac{Vhtb^2}{4I}}\right]$$

$$= \frac{1}{2AGt}\left[q_0(2h+2b) - \frac{Vhtb}{2I}(h+b)\right]$$

$$\beta = \frac{1}{2AGt}(h+b)\left[2q_0 - \frac{Vhtb}{2I}\right]$$

Since $\beta = 0$; $\quad 2q_0 - \dfrac{Vhtb}{2I} = 0$

$$\boxed{q_0 = \frac{Vhtb}{4I}}$$

$$\therefore\ q_{AB} = q_0 + \frac{Vt}{2I}(hs - s^2) = \frac{Vhtb}{4I} + \frac{Vt}{2I}(hs - s^2)$$

$$q_{BC} = q_0 - \frac{Vhts}{2I} = \frac{Vhtb}{4I} - \frac{Vhts}{2I}$$

$$q_{CD} = q_0 - \frac{Vhtb}{2I} - \frac{Vt}{2I}(hs - s^2) = \frac{Vhtb}{4I} - \frac{Vhtb}{2I} - \frac{Vt}{2I}(hs - s^2)$$

$$q_{DA} = q_0 - \frac{Vhtb}{2I} + \frac{Vhts}{2I} = \frac{Vhtb}{4I} - \frac{Vhtb}{2I} + \frac{Vhts}{2I}$$

$$q_{AB} = \frac{Vhtb}{4I} + \frac{Vt}{2I}(hs - s^2)$$

@ A ; $s=0$ ; $q_A = \frac{Vhtb}{4I}$

@ B ; $s=h$ ; $q_B = \frac{Vhtb}{4I}$

@ E ; $s=\frac{h}{2}$ ; $q_E = \frac{Vhtb}{4I} + \frac{Vt}{2I}\left(\frac{h^2}{2}-\frac{h^2}{4}\right)$

$$= \frac{Vhtb}{4I} + \frac{Vh^2t}{8I}$$

$q_{BC} = \frac{Vhtb}{4I} - \frac{Vhts}{2I}$

@ B ; $s=0$ ; $q_B = \frac{Vhtb}{4I}$ ;

@ C ; $s=b$ ; $q_C = \frac{Vhtb}{4I} - \frac{Vhtb}{2I} = \frac{-Vhtb}{4I}$

@ G ; $s=b/2$ ; $q_G = \frac{Vhtb}{4I} - \frac{Vhtb}{4I} = 0.$

$q_{CD} = \frac{-Vhtb}{4I} - \frac{Vt}{2I}\left(hs-s^2\right)$

@ C ; $s=0$ ; $q_C = \frac{-Vhtb}{4I}$

@ D ; $s=h$ ; $q_D = \frac{-Vhtb}{4I}$

@ F ; $s=h/2$ ; $q_F = \frac{-Vhtb}{4I} - \frac{Vt}{2I}\left(\frac{h^2}{2}-\frac{h^2}{4}\right)$

$$= \frac{-Vhtb}{4I} - \frac{Vh^2t}{8I}$$

$q_{DA} = \frac{-Vhtb}{4I} + \frac{Vhts}{2I}$

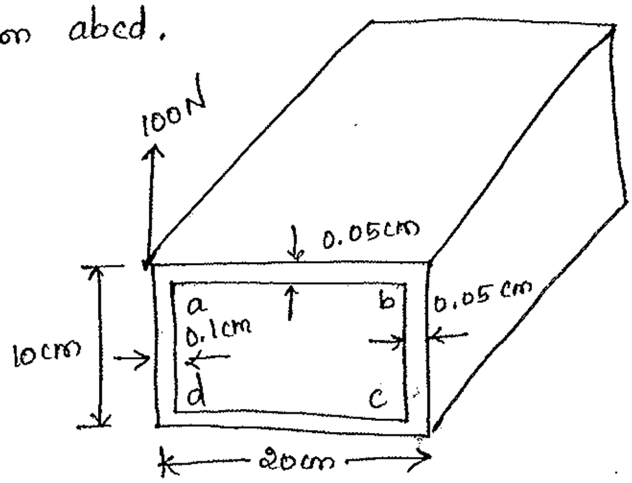
@ D ; $s=0$ ; $q_D = \frac{-Vhtb}{4I}$

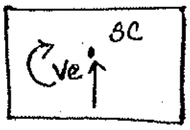
@ A ; $s=b$ ; $q_A = \frac{-Vhtb}{4I} + \frac{Vhtb}{2I} = \frac{Vhtb}{4I}$

@ H ; $s=b/2$ ; $q_H = \frac{-Vhtb}{4I} + \frac{Vhtb}{4I} = 0.$

Problem : 9

Figure shows a single cell rectangular beam carrying the load of 100 N as shown. Find the internal resisting shear flow pattern at section abcd.



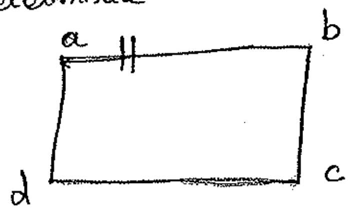




$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{[(20 \times 0.05) \times 10]2 + (10 \times 0.05)20}{(10 \times 0.1) + [(20 \times 0.05)]2 + 10 \times 0.05} = 8.57 \text{ cm}$$

$$I_{xx} = \Sigma Ay^2 = \frac{0.1 \times 10^3}{12} + \frac{0.05 \times 10^3}{12} + \left\{ \left[ \frac{20 \times 0.05^3}{12} + \left[ 20 \times 0.05 \times 5^2 \right] \right] \right\}$$

$$= 62.5 \text{ cm}^4$$

cut and open the tube all along the generator at some point to make determinate




$$q = \frac{-V}{I} \int y t ds.$$

$$q_{AB} = \frac{-100}{62.5} \int 5(0.05) ds = -0.4s$$

@ A ; $s = 0$ ; $q_A = 0$

@ B ; $s = 20$ ; $q_B = -8$ N/cm

$$q_{BC} = q_B - \frac{100}{62.5} \int (5-s)(0.05) ds$$

$$= -8 - 0.08 \left[ 5s - \frac{s^2}{2} \right]$$

at B ; $S=0$ ; $q_B = -8\ N/cm$

@ C ; $S=10$ ; $q_c = -8\ N/cm$

$$q_{CD} = q_c - \frac{100}{62.5}\int(-5)(0.05)\,ds = -8 + 0.4S$$

@ C ; $S=0$ ; $q_c = -8\ N/cm$

@ D ; $S=20$ ; $q_D = 0\ N/cm$

$$q_{DA} = q_D - \frac{100}{62.5}\int -(5-S)(0.1)\,ds = 0.16\left[5S - \frac{S^2}{2}\right]$$

$$= 0.16\left[5S - \frac{S^2}{2}\right]$$

Let $q_0$ be the indeterminate shear existing at the cut before cutting

∴ For uncut section,

$$q_{AB} = q_0 - 0.4S$$

$$q_{BC} = q_0 - 8 - 0.08\left[5S - \frac{S^2}{2}\right]$$

$$q_{CD} = q_0 - 8 + 0.4S$$

$$q_{DA} = q_0 + 0.16\left[5S - \frac{S^2}{2}\right]$$

For shear flow due to bending, let us assume that the load passes through shear centre and hence $\beta = 0$

$$\beta = \frac{q}{2AG}\oint\frac{ds}{t} = \frac{1}{2AG}\oint\frac{q\cdot ds}{t}$$

$$\therefore \beta = \frac{1}{2AG}\left[\int_0^{20}\frac{q_0 - 0.4S}{0.05} + \int_0^{10}\frac{q_0 - 8 - 0.08\left(5S - \frac{S^2}{2}\right)}{0.05} + \int_0^{20}\frac{q_0 - 8 + 0.4S}{0.05} + \int_0^{10}\frac{q_0 + 0.16\left(5S - \frac{S^2}{2}\right)}{0.1}\,ds\right.$$

$$= \frac{1}{2AG}\left[\frac{1}{0.05}\left[q_0 s - \frac{0.4s^2}{2}\right]_0^{20} + \frac{1}{0.05}\left[q_0 s - 8s - 0.08\left(\frac{5s^2}{2} - \frac{s^3}{6}\right)\right]_0^{10}\right.$$

$$\left. + \frac{1}{0.05}\left[q_0 s - 8s - \frac{0.4s^2}{2}\right]_0^{20} + \frac{1}{0.1}\left[q_0 s + 0.16\left(\frac{5s^2}{2} - \frac{s^3}{6}\right)_0^{10}\right]\right]$$

$$= \frac{1}{2AG}\left[400q_0 + 200q_0 - 1600 + 400q_0 - 3200 + 100q_0\right]$$

$$\beta = \frac{1}{2AG}\left[1100q_0 - 4800\right]$$

Since $\beta = 0$ ; $1100q_0 - 4800 = 0$

$$\therefore \boxed{q_0 = 4.3636 \text{ N/cm}}$$

$\therefore q_{AB} = q_0 - 0.4s = 4.3636 - 0.4s$

$q_{BC} = q_0 - 8 - 0.08\left(5s - \frac{s^2}{2}\right) = 4.3636 - 8 - 0.08\left(5s - \frac{s^2}{2}\right)$

$q_{CD} = q_0 - 8 + 0.4s = 4.3636 - 8 + 0.4s$

$q_{DA} = q_0 + 0.16\left(5s - \frac{s^2}{2}\right) = 4.3636 + 0.16\left(5s - \frac{s^2}{2}\right)$

$\underline{\underline{q_{AB}}}$

At A ; $s=0$ ; $q_A = 4.3636$ N/cm

At B ; $s=20$ ; $q_B = -3.6364$ N/cm

Equate $q_{AB} = 0$.

(ie) $q_{AB} = q_0 - 0.4s = 0$.

$4.3636 - 0.4s = 0$

$\boxed{s = 10.9 \text{ cm}}$

$\underline{\underline{q_{BC}}}$ At B ; $s=0$ ; $q_B = -3.6364$ N/cm

At C ; $s=10$ ; $q_C = -3.6364$ N/cm

At mid of BC ; $s=5$ ; $\therefore$

$q_F = -4.6364$ N/cm

$\underline{\underline{q_{CD}}}$

at C ; $s=0$ ; $q_C = -3.6364$ N/cm

at D ; $s=20$ ; $q_D = 4.3636$ N/cm

Equating $q_{CD} = 0$

$4.3636 - 8 + 0.4s = 0$

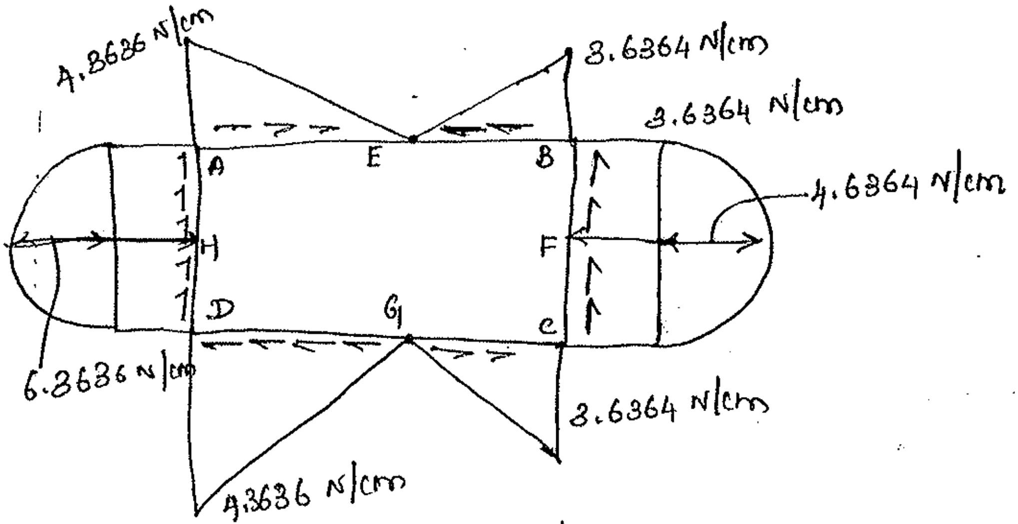
$\boxed{s = 9.1 \text{ cm}}$

$\underline{\underline{q_{DA}}}$

At D ; $s=0$ ; $q_D = 4.3636$ N/cm

At A ; $s=10$ ; $q_A = 4.3636$ N/cm

At mid DA ; $s=5$ ; $q_H = 6.3636$ N/cm




To find the position of Shear centre :-

Taking moment about D

$$V \times e = (3.6364 \times 10 \times 20) + \left[\frac{2}{3}(4.6364 - 3.6364) \times 10 \times 20\right] + \left(\frac{3.6364}{2} \times 9.1 \times 10\right) - \left(\frac{4.3636}{2} \times 10.9 \times 10\right)$$

and V = 100 N

$$\boxed{e = 7.88\,cm}$$

Torque $T = V \times e = 100 \times 7.88 = 788$ N-cm

Shear flow due to torque.

$$q_t = \frac{T}{2A} = \frac{788}{2 \times 10 \times 20} = 1.97\ N/cm$$

$\therefore \boxed{q_t = 1.97\ N/cm} \rightarrow$ (clockwise, $\because$ +ve)

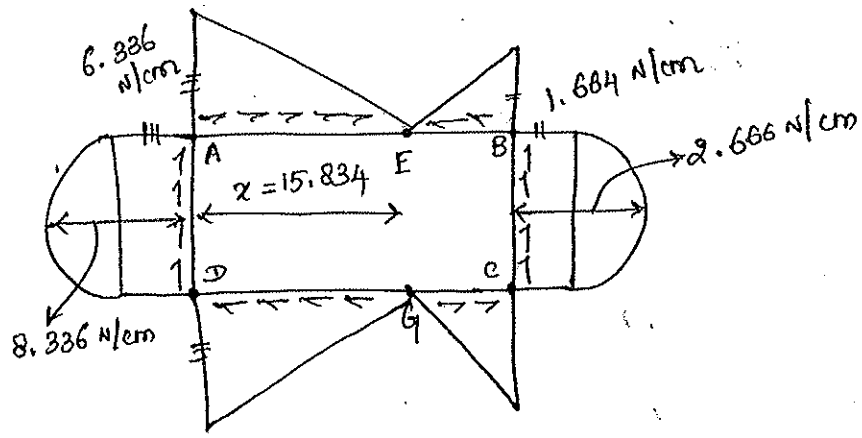
Now Adding this shear flow to the Shear flow due to bending, we get a final Shear flow diagram.

$$q_{A\,net} = 4.3636 + 1.97 = 6.3336\ N/cm$$
$$q_{B\,net} = -3.6364 + 1.97 = -1.6666\ N/cm$$
$$q_{F\,net} = -4.6364 + 1.97 = -2.6664\ N/cm$$

$q_{C\,net} = -3.6364 + 1.97 = -1.6664 \text{ N/cm}$

$q_{H\,net} = 6.3636 + 1.97 = 8.336 \text{ N/cm}$

$q_{D\,net} = 4.3636 + 1.97 = 6.3336 \text{ N/cm}$




To find the position of E (or) G:

By Equilateral triangle;

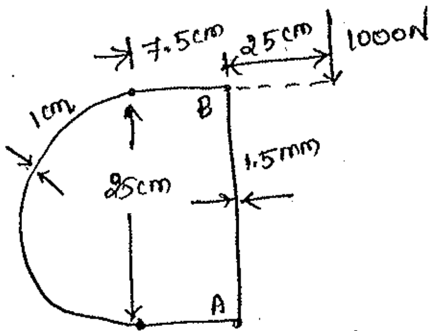
$$\frac{x}{6.336} = \frac{20-x}{1.664}$$

$$1.6664x = (20-x)\,6.3336$$

$$8x = 126.672 \quad ; \quad \boxed{x = 15.834 \text{ cm}}$$

Problem: 10

Determine the Shear flow and Shear centre for the section shown in figure.




Let $q_1$ and $q_2$ be the constant shear flow due to the load as shown in figure

$$= \frac{1}{2AG}\left[\frac{1}{0.05}\left[q_0 s - \frac{0.4s^2}{2}\right]_0^{20} + \frac{1}{0.05}\left[q_0 s - 8s - 0.08\left(\frac{5s^2}{2} - \frac{s^3}{6}\right)\right]_0^{10} + \frac{1}{0.05}\left[q_0 s - 8s - \frac{0.4s^2}{2}\right]_0^{20} + \frac{1}{0.1}\left[q_0 s + 0.16\left(\frac{5s^2}{2} - \frac{s^3}{6}\right)\right]_0^{10}\right]$$

$$= \frac{1}{2AG}\left[400q_0 + 200q_0 - 1600 + 400q_0 - 3200 + 100q_0\right]$$

$$\beta = \frac{1}{2AG}\left[1100q_0 - 4800\right]$$

Since $\beta = 0$ ; $1100q_0 - 4800 = 0$

$$\therefore \boxed{q_0 = 4.3636 \text{ N/cm}}$$

$\therefore q_{AB} = q_0 - 0.4s = 4.3636 - 0.4s$

$q_{BC} = q_0 - 8 - 0.08\left(5s - \frac{s^2}{2}\right) = 4.3636 - 8 - 0.08\left(5s - \frac{s^2}{2}\right)$

$q_{CD} = q_0 - 8 + 0.4s = 4.3636 - 8 + 0.4s$

$q_{DA} = q_0 + 0.16\left(5s - \frac{s^2}{2}\right) = 4.3636 + 0.16\left(5s - \frac{s^2}{2}\right)$

$q_{AB}$

At A ; $s=0$ ; $q_A = 4.3636$ N/cm

At B ; $s=20$ ; $q_B = -3.6364$ N/cm

To Equate $q_{AB} = 0$.

(ie) $q_{AB} = q_0 - 0.4s = 0$.

$4.3636 - 0.4s = 0$

$\boxed{s = 10.9 \text{ cm}}$

$q_{BC}$ At B ; $s=0$ ; $q_B = -3.6364$ N/cm

At C ; $s=10$ ; $q_C = -3.6364$ N/cm

At mid of BC ; $s=5$ ; $\therefore$

$q_F = -4.6364$ N/cm

$q_{CD}$

at C ; $s=0$ ; $q_C = -3.6364$ N/cm

at D ; $s=20$ ; $q_D = 4.3636$ N/cm

Equating $q_{CD} = 0$

$4.3636 - 8 + 0.4s = 0$

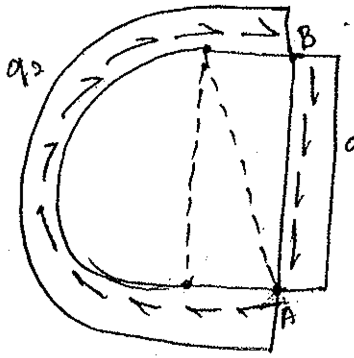
$\boxed{s = 9.1 \text{ cm}}$

$q_{DA}$

At D ; $s=0$ ; $q_D = 4.3636$ N/cm

At A ; $s=10$ ; $q_A = 4.3636$ N/cm

At mid DA ; $s=5$ ; $q_H = 6.3636$ N/cm




Taking moment about point A

$$\left(q_2 \times 7.5 \times 25\right) + q_2 \times 2\left[\frac{\pi \times 12.5^2}{2} + \frac{7.5 \times 25}{2}\right]$$

$$= 1000 \times 25$$

$$865.818\, q_2 = 25000$$

$$\boxed{q_2 = 28.8725 \text{ N/cm}}$$

By force Equilibrium,

$$q_1 \times 25 - q_2 \times 25 = 1000$$

$$25 q_1 = 1000 + 25 q_2$$

$$= 1000 + 25\,(28.8725)$$

$$\boxed{q_1 = 68.8725 \text{ N/cm}}$$

To locate the Shear centre:

Subtract the value of $q_t$ from $q_1$ and $q_2$ considering that the cell will produce zero twist

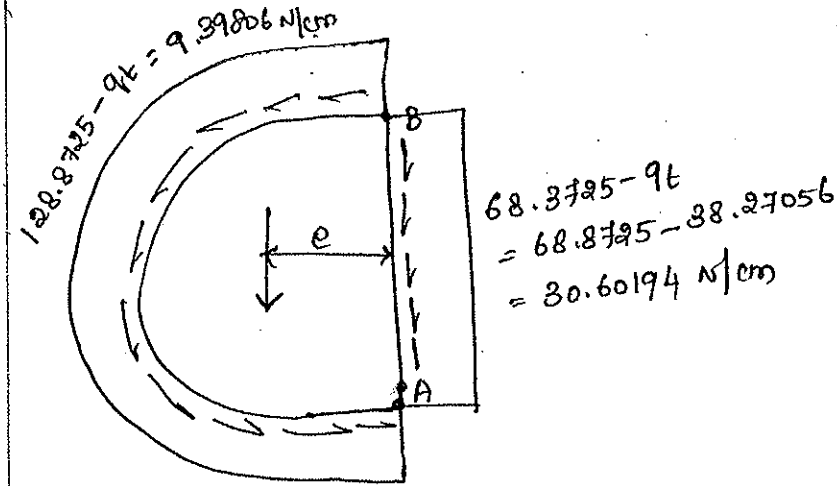
$$\beta = \frac{1}{2AG}\oint \frac{q\,ds}{t} = 0$$

$$\frac{1}{2AG}\left[\int^{(7.5\times 2)+\pi(12.5)} \frac{28.8725 - q_t}{0.1}\, ds + \int_0^{25} \frac{68.8725 - q_t}{0.15}\, ds\right] = 0$$

$$\left[\frac{28.8725 - q_t}{0.1}\right] \times 54.2699 + \left[\frac{68.8725 - q_t}{0.15}\right] \times 25 = 0$$

$$709.3657\, q_t = 27147.82688$$

$$\boxed{q_t = 38.27056 \text{ N/cm}}$$



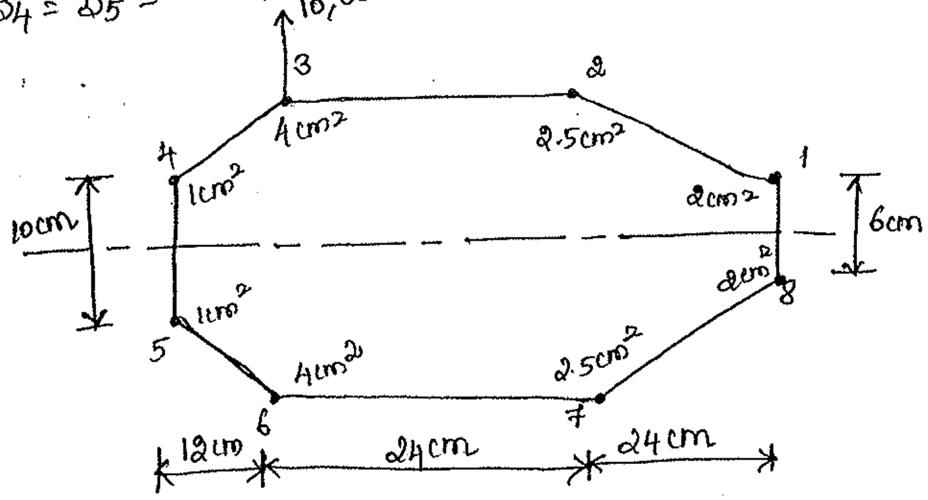

Taking moment about A ;

$$1000 \times e = 9.39806 \times 2 \times \left[ \frac{\pi \times 12.5^2}{2} + \frac{7.5 \times 25}{2} \right] + 9.39806 \times (7.5 \times 25)$$

$$\boxed{e = 8.1375 \text{ cm}}$$

Problem : 11

The idealized single cell thin walled tube is shown in figure has a horizontal axis of symmetry. Direct stresses are carried by the booms $B_1$ to $B_8$ ; while the walls are effective only in carrying the shear stress. Calculate the distribution of shear flow around the section. Given $B_1 = B_8 = 2 \text{ cm}^2$ ; $B_2 = B_7 = 2.5 \text{ cm}^2$ ; $B_3 = B_6 = 4 \text{ cm}^2$ ; $B_4 = B_5 = 1 \text{ cm}^2$.



$$I_{xx} = \Sigma A y^2 = [2 \times 3^2]2 + [2.5 \times 10^2]2 + [4 \times 10^2]2 + [1 \times 5^2]2 = 1386\ cm^4.$$

Cut and open the section at 1-2 to make the section determinate

$$\left(q = -\frac{V}{I} \Sigma A_i y_i\right)$$

$$q_{12} = \frac{-10000}{1386}(2 \times 6) = -43.29\ N/cm$$

$$q_{23} = -43.29 - \frac{10000}{1386}(2.5 \times 10) = -223.665\ N/cm$$

$$q_{34} = -223.665 - \frac{10000}{1386}(4 \times 10) = -512.265\ N/cm$$

$$q_{45} = -512.265 - \frac{10000}{1386}(1 \times 5) = -548.34\ N/cm$$

$$q_{67} = -512.265 - \frac{10000}{1386}(4 \times -10) = -223.665\ N/cm$$

$$q_{78} = -223.665 - \frac{10000}{1386}(25 \times -30) = -43.29\ N/cm$$

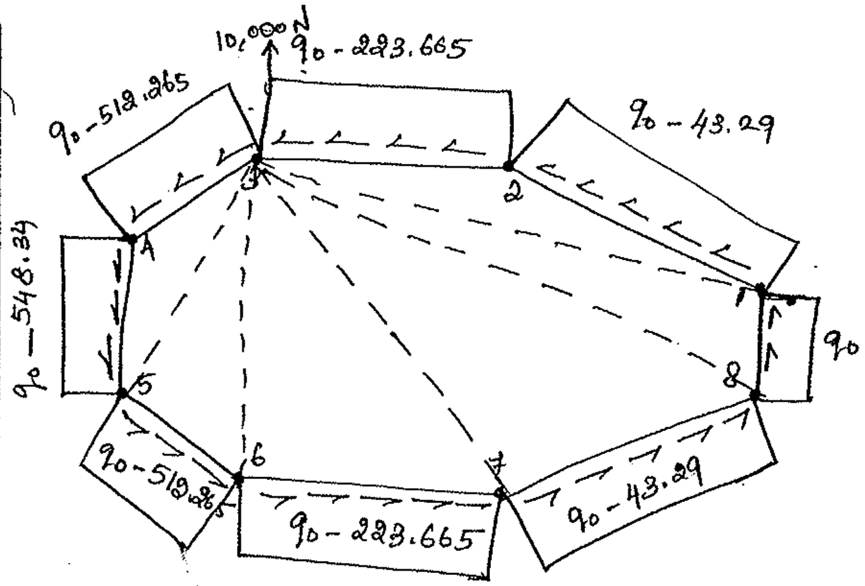
$$q_{81} = -43.29 - \frac{10000}{1386}(2 \times -3) = 0$$

Let $q_0$ be the indeterminate shear flow existing at the cut before cutting.

∴ For uncut section,

| | |
|---|---|
| $q_{12} = q_0 - 43.29$ | $q_{56} = q_0 - 512.265$ |
| $q_{23} = q_0 - 223.665$ | $q_{67} = q_0 - 223.665$ |
| $q_{34} = q_0 - 512.265$ | $q_{78} = q_0 - 43.29$ |
| $q_{45} = q_0 - 548.34$ | $q_{81} = q_0$ |

Now draw the shear flow diagram.




$$\text{Area of } \Delta 453 = \frac{1}{2} \times 10 \times 12 = 60\,cm^2$$

$$\text{Area of } \Delta 536 = \frac{1}{2} \times 20 \times 12 = 120\,cm^2$$

$$\text{Area of } \Delta 367 = \frac{1}{2} \times 24 \times 20 = 240\,cm^2$$

$$\text{Area of } \Delta 183 = \frac{1}{2} \times 6 \times 48 = 144\,cm^2$$

$$\text{Area of } \Delta 123 = \frac{1}{2} \times 24 \times (10-3) = 84\,cm^2$$

$$\text{Area of } \Delta 378 = (\text{Area of } 123781) - \text{Area of } 123 - \text{Area of } 183$$

$$= [\text{Area of } 2372 + \text{Area of } 78127] - [\text{Area of } 123] - [\text{Area of } 183]$$

$$= \left[\frac{1}{2} \times 24 \times 20 + \left(\frac{6+20}{2}\right) 24\right] - 84 - 144$$

$$= 240 + 312 - 84 - 144$$

$$= 324\,cm^4$$

Taking moment about point 3 ;

$$q_{45} \times 2 \times \Delta 453 + q_{56} \times 2 \times \Delta 536 + q_{67} \times 2 \times \Delta 367 + q_{78} \times 2 \times \Delta 378 + q_{81} \times 2 \times \Delta 183 + q_{12} \times 2 \times \Delta 123 = 0$$

$$(q_0 - 548.34) 2 \times 60 + (q_0 - 512.265) 2 \times 120 + (q_0 - 223.665) 2 \times 240 + (q_0 - 43.29) \times 2 \times 324 + (q_0 \times 2 \times 144) + (q_0 - 43.29) \times 2 \times 84 = 0$$

$1944\,q_0 = 331428.24$

$$\boxed{q_0 = 170.4877 \text{ N/cm}}$$

$q_{12} = 170.4877 - 43.29 = 127.1977$ N/cm

$q_{23} = 170.4877 - 223.665 = -53.1773$ N/cm

$q_{34} = 170.4877 - 512.265 = -341.7773$ N/cm

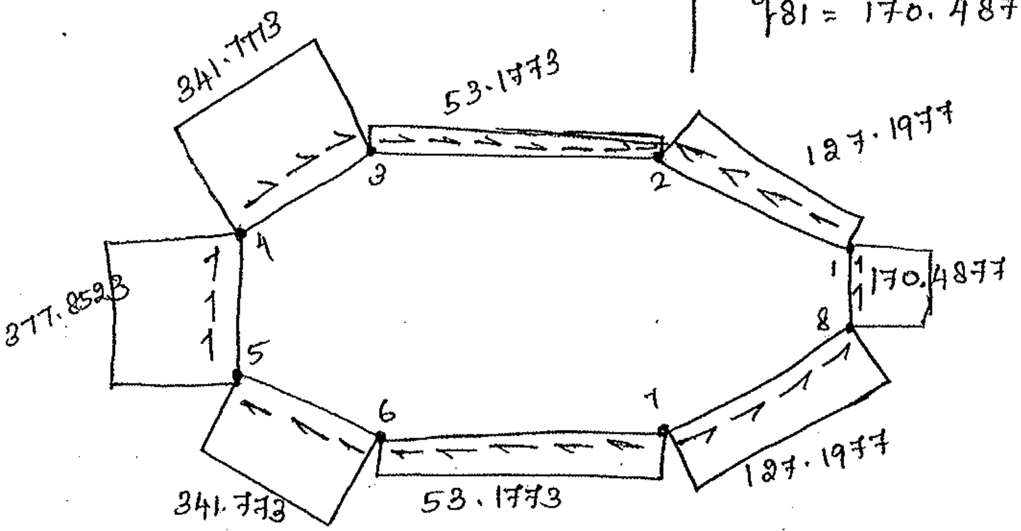
$q_{45} = 170.4877 - 548.34$
$= -377.8523$ N/cm

$q_{56} = 170.4877 - 512.265$
$= -341.7773$ N/cm

$q_{67} = 170.4877 - 223.665$
$= -53.17773$ N/cm

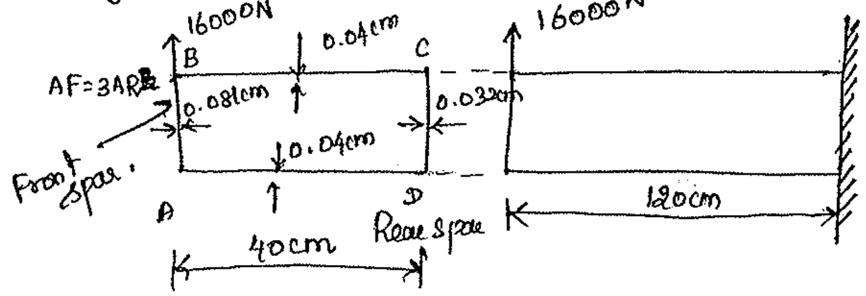
$q_{18} = 170.4877 - 43.29$
$= 127.1977$ N/cm

$q_{81} = 170.4877$ N/cm.

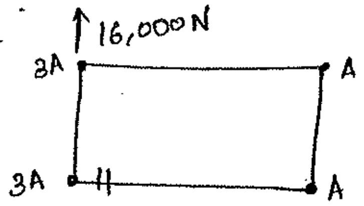



Problem: 12

The box beam is shown in figure has a front spar flange area of which are three times the rear spar flange areas. Find the angle of twist at free end. Assume $G = 4\times10^6$ N/cm$^2$.



Consider




$AF = 3AR$

$$I = \left[A \times 5^2 + (3A)5^2\right] \times 2 = 200A.$$

For Shear flow due to bending, cut and open the section at A,

$$q_{AB} = \frac{-16{,}000}{200A}(3A)(-5) = 1200\ N/cm$$

$$q_{BC} = 1200 - \frac{16000}{200A}(3A)(5) = 0$$

$$q_{CD} = -\frac{16000}{200A}(A)(5) = -400\ N/cm$$

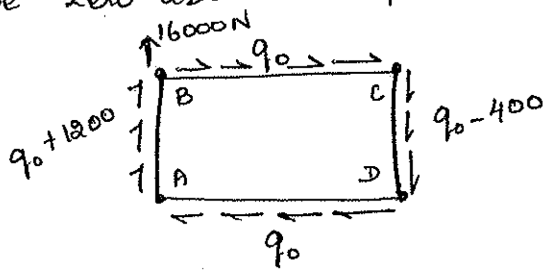
$$q_{DA} = -400 - \frac{16000}{200A}(A)(-5) = 0.$$

Let $q_0$ be the indeterminate value existing at the cut due to bending.

$q_{AB} = q_0 + 1200$ | $q_{CD} = q_0 - 400$

$q_{BC} = q_0$ | $q_{DA} = q_0$

$q_0$ be such that, the moment due to the internal shear flow must be zero about the point on the load line / line of load.




Taking moment about A,

$$(q_0 \times 40 \times 10) + (q_0 - 400)(10)(40) = 0$$

$$800q_0 = 160000$$

$$q_0 = 200\ N/cm$$

Taking moment about A,

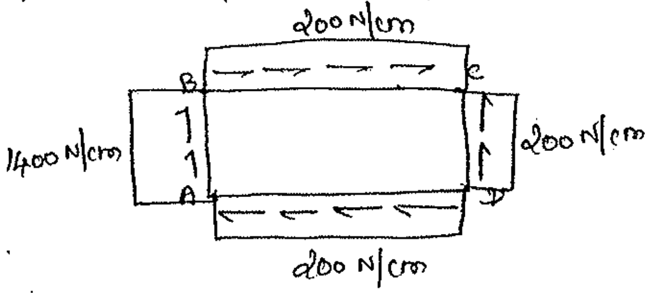
$$q_0 \times 40 \times 10 + (q_0 - 400)(10)(40) = 0$$

$$800 q_0 = 160000$$

$$q_0 = 200 \text{ N/cm}$$

$\therefore$ $q_{AB} = 1400$ N/cm $\qquad q_{CD} = -200$ N/cm

$q_{BC} = 200$ N/cm $\qquad q_{DA} = 200$ N/cm




Twist per unit length,

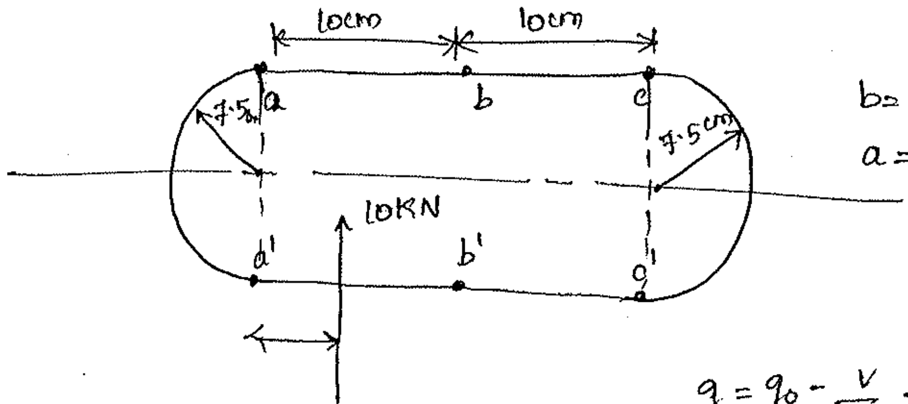
$$\beta = \frac{1}{2AG} \oint \frac{q\,ds}{t}$$

$$= \frac{1}{2 \times 40 \times 10 \times 4 \times 10^6} \left[ \frac{1400 \times 10}{0.081} + \frac{200 \times 40}{0.04} + \frac{200 \times 10}{0.032} + \frac{200 \times 40}{0.04} \right]$$

$$= 1.1086 \times 10^{-4} \text{ rad/cm}.$$

Total angle of twist $\phi = 120\beta = 0.0133$ rad/cm.

Problem. 13 Find the shear flow for the closed tube shown in fig.




$b = b' = 2\text{cm}^2$

$a = a' = c = c' = 1\text{cm}$

$$q = q_0 - \frac{V}{I} \Sigma A y^2.$$

$$I = (1\times 7.5^2)4 + (2\times 7.5^2)\times 2$$
$$= 450\ cm^4$$

cut and open the Section

$$q_{ab} = \frac{-10}{450}(1)(7.5) = -0.1667\ KN/cm$$

$$q_{bc} = -0.1667 - \frac{10}{450}(2)(7.5) = -0.5\ KN/cm$$

$$q_{cc'} = -0.5 - \frac{10}{450}(1)\times 7.5 = -0.6667\ KN/cm$$

$$q_{c'b'} = -0.667 - \frac{10}{450}(1)(-7.5) = -0.5\ KN/cm$$

$$q_{b'a'} = -0.5 - \frac{10}{450}(2)(-7.5) = -0.1667\ KN/cm.$$

$$q_{a'a} = -0.1667 - \frac{10}{450}(1)(-7.5) = 0.$$

Let $q_0$ be the indeterminate shear flow at the cut,

$q_{ab} = q_0 - 0.1667.$

$q_{bc} = q_0 - 0.5$

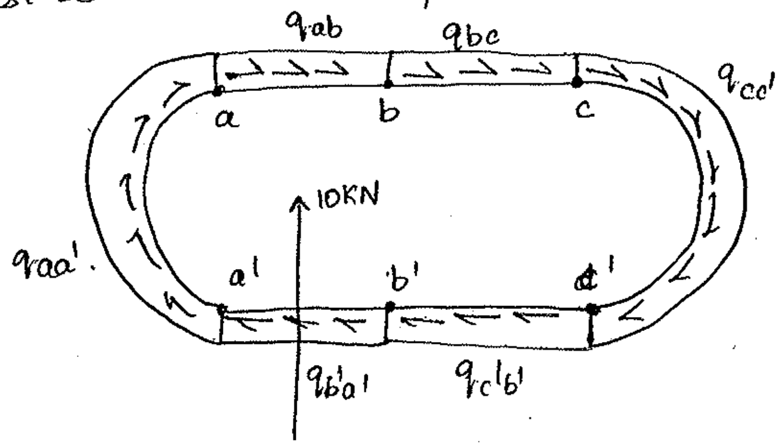
$q_{cc'} = q_0 - 0.6667$

$q_{c'b'} = q_0 - 0.5$

$q_{b'a'} = q_0 - 0.1667$

$q_{a'a} = q_0$

$q_0$ be such that, the moment due to the internal shear flow must be zero about a point on the load line.



Taking moment about intersection of load line and bottom web

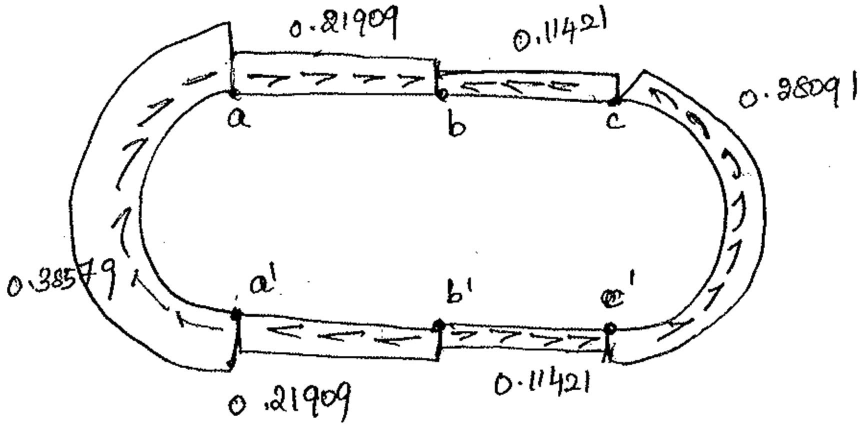
$$q_0 \times 2\left[\frac{\pi \times 7.5^2}{2} + \frac{5 \times 15}{2}\right] + \left[q_0 - 0.1667\right] 2 \times \left(\frac{10 \times 15}{2}\right)$$

$$+ (q_0 - 0.5) \times 2 \times \left(\frac{10 \times 15}{12}\right) + (q_0 - 0.667) 2\left[\frac{\pi \times 15^2}{2} + \frac{15 \times 15}{2}\right] = 0$$

$$953.4291735\, q_0 = 367.828115$$

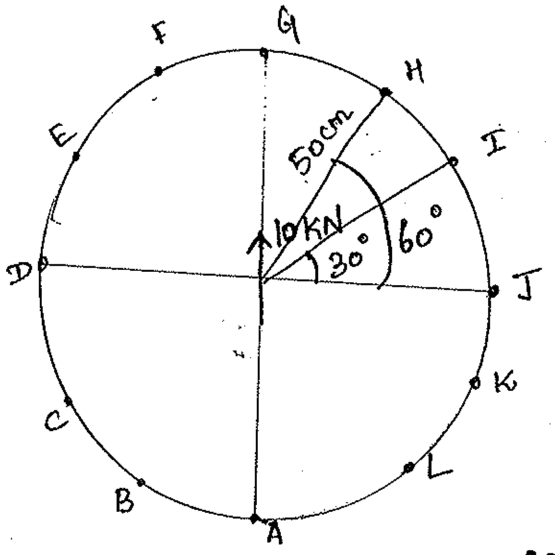
$$q_0 = 0.385.79 \text{ KN/cm}$$

| | |
|---|---|
| $q_{ab} = 0.21909$ KN/cm | $q_{c'b'} = -0.11421$ KN/cm |
| $q_{bc} = -0.11421$ KN/cm | $q_{b'a'} = 0.21909$ KN/cm |
| $q_{cc'} = -0.28091$ KN/cm | $q_{a'a} = 0.38579$ KN/cm |




Problem: 14

A Fuselage bulkhead of 1m diameter, has a Stringers equally placed around the Section Starting from top point. Each Stringer area is 6.25 cm². The bulkhead is Subjected to a Symmetrical vertical Shear load of 10 KN. Find the Shear flow around the bulk head.




$$I = (6.25 \times 50^2) \times 2 + [6.25 \times (50 \sin 30°)^2] 4 + [6.25 \times (50 \times \sin 60)^2] \times 4$$

$$\boxed{I_{xx} = 93750 \text{ cm}^4}$$

$$q_{AB} = \frac{-10}{93750} \times 6.25 \times (-50) = 0.03333 \text{ KN/cm}$$

$$q_{BC} = 0.03333 - \frac{10}{93750} \times 6.25 \times (-50 \sin 60°) = 0.0622 \text{ KN/cm}$$

$$q_{CD} = 0.0622 - \frac{10}{93750} \times 6.25 \times (-50 \sin 30°) = -0.078867 \text{ KN/cm}$$

$$q_{DE} = -0.078867 - \frac{10}{93750} \times 6.25 \times 0 = -0.078867 \text{ KN/cm}$$

$$q_{EF} = 0.078867 - \frac{10}{93750} \times 6.25 \times (50 \sin 30°) = 0.0622 \text{ KN/cm}$$

$$q_{FG} = 0.0622 - \frac{10}{93750} \times 6.25 \times (50 \sin 60°) = 0.03333 \text{ KN/cm}$$

$$q_{GH} = 0.03333 - \frac{10}{93750} \times 6.25 \times 50 = 0.$$

$$q_{HI} = 0 - \frac{10}{93750} \times 6.25 \times 50 \sin 60^\circ = -0.028867 \text{ KN/cm}$$

$$q_{IJ} = -0.028867 - \frac{10}{93750} \times 6.25 \times 50 \sin 30 = -0.045534 \text{ KN/cm}$$

$$q_{JK} = -0.045534 - \frac{10}{93750} \times 6.25 \times 0 = -0.045534 \text{ KN/cm}$$

$$q_{KL} = -0.045534 - \frac{10}{93750} \times 6.25 \times (-50 \sin 30^\circ) = -0.028867 \text{ KN/cm}$$

$$q_{LA} = -0.028867 - \frac{10}{93750} \times 6.25 \times (-50 \sin 60^\circ) = 0$$

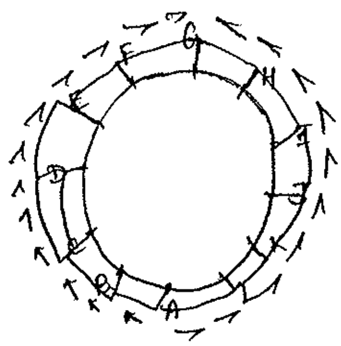
Let $q_0$ be the indeterminate shear flow at the cut.

Add $q_0$ value to all "$q$" values

Since $\beta = 0$; $\frac{1}{2AGt} \int q\,ds = 0 \Rightarrow q_s = 0$; $q = 0$

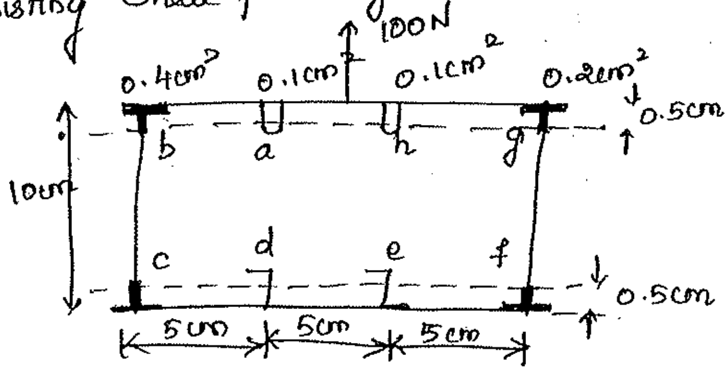
$$12 q_0 + 0.2 = 0 \Rightarrow q_0 = -0.016667 \text{ KN/cm}$$

| | |
|---|---|
| $q_{AB} = 0.01666$ KN/cm | $q_{GH} = -0.01666$ KN/cm |
| $q_{BC} = 0.04553$ KN/cm | $q_{HI} = -0.04553$ KN/cm |
| $q_{CD} = 0.06220$ KN/cm | $q_{IJ} = -0.06220$ KN/cm |
| $q_{DE} = 0.06220$ KN/cm | $q_{JK} = -0.06220$ KN/cm |
| $q_{EF} = 0.04553$ KN/cm | $q_{KL} = -0.04553$ KN/cm |
| $q_{FG} = 0.01666$ KN/cm | $q_{LA} = -0.01666$ KN/cm. |



Problem : 15

Figure carries a load of 100 N Shear load. Calculate the resisting Shear flow system.




$$q = q_0 - \frac{V}{I}\sum Ay$$

$$I = [0.4 \times 5^2] \times 2 + [0.1 \times 5^2]4 + [0.2 \times 5^2] \times 2 = 40\,cm^4$$

Cut and open the section between a and h.

$$\frac{V}{I} = \frac{100}{40} = 2.5\ N/cm^4.$$

$$q_{AB} = -2.5 \times 0.1 \times 5 = -1.25\ N/cm$$

$$q_{BC} = -1.25 - 2.5 \times 0.4 \times 5 = -6.25\ N/cm$$

$$q_{CD} = -6.25 - 2.5 \times 0.4 \times (-5) = -1.25\ N/cm$$

$$q_{DE} = -1.25 - 2.5 \times 0.1 \times (-5) = 0.$$

$$q_{EF} = 0 - 2.5 \times 0.1 \times (-5) = 1.25\ N/cm$$

$$q_{FG} = 1.25 - 2.5 \times 0.2 \times (-5) = 3.75\ N/cm$$

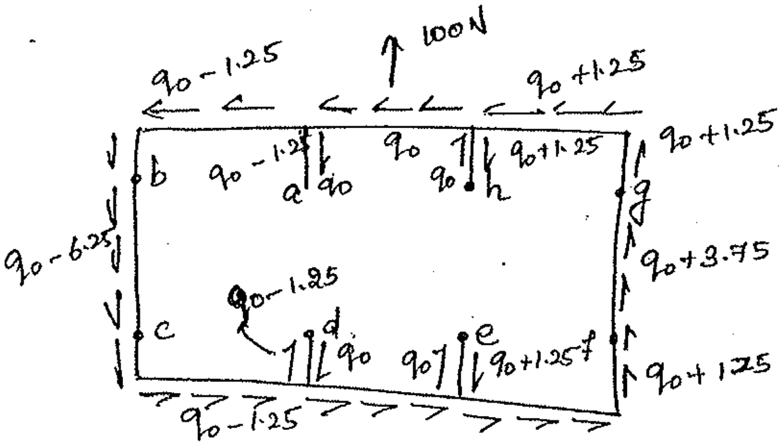
$$q_{GH} = 3.75 - 2.5 \times 0.2 \times (5) = 1.25\ N/cm$$

$$q_{HA} = 1.25 - 2.5 \times 0.1 \times 5 = 0.$$

Let $q_0$ be the indeterminate shear flow at the cut

$\therefore$ For uncut section.

| | |
|---|---|
| $q_{AB} = q_0 - 1.25$ | $q_{EF} = q_0 + 1.25$ |
| $q_{BC} = q_0 - 6.25$ | $q_{FG} = q_0 + 3.75$ |
| $q_{CD} = q_0 - 1.25$ | $q_{GH} = q_0 + 1.25$ |
| $q_{DE} = q_0$ | $q_{HA} = q_0.$ |




Taking moment about lower left corner,

$$(q_0-1.25)\times 5\times 11 + q_0\times 5\times 11 + (q_0\times 1.25)\times 5\times 11 + (q_0+1.25)(0.5\times 15)$$
$$+ (q_0+3.75)10\times 15 + (q_0+1.25)(0.5\times 15)$$
$$-1.25\times(0.5\times 5)\times 2 - 1.25\times(0.5\times 10)\times 2 = 750$$

$$330\, q_0 = 187.5$$

$$\boxed{q_0 = 0.568\ N/cm}$$

$\therefore$ $q_{AB} = -0.682$ N/cm

$q_{BC} = -5.682$ N/cm

$q_{CD} = -0.682$ N/cm

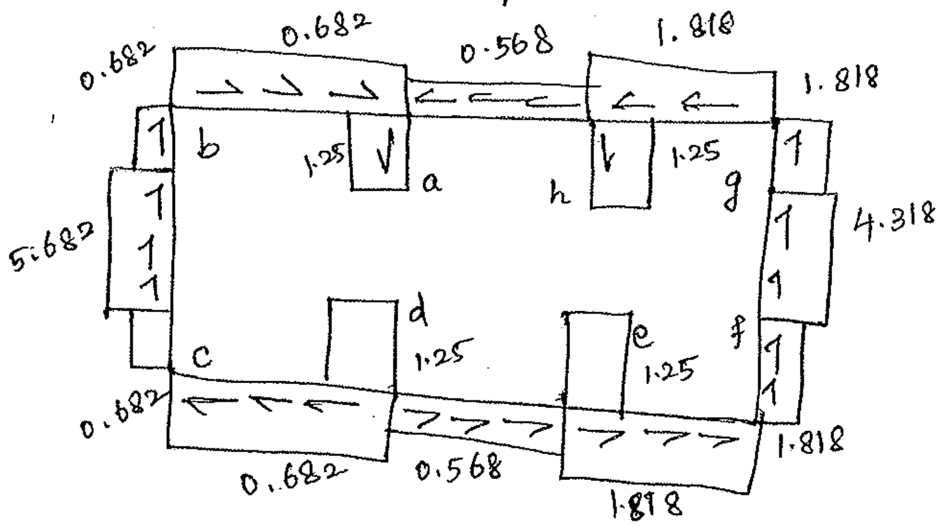
$q_{DE} = 0.568$ N/cm

$q_{EF} = 1.818$ N/cm

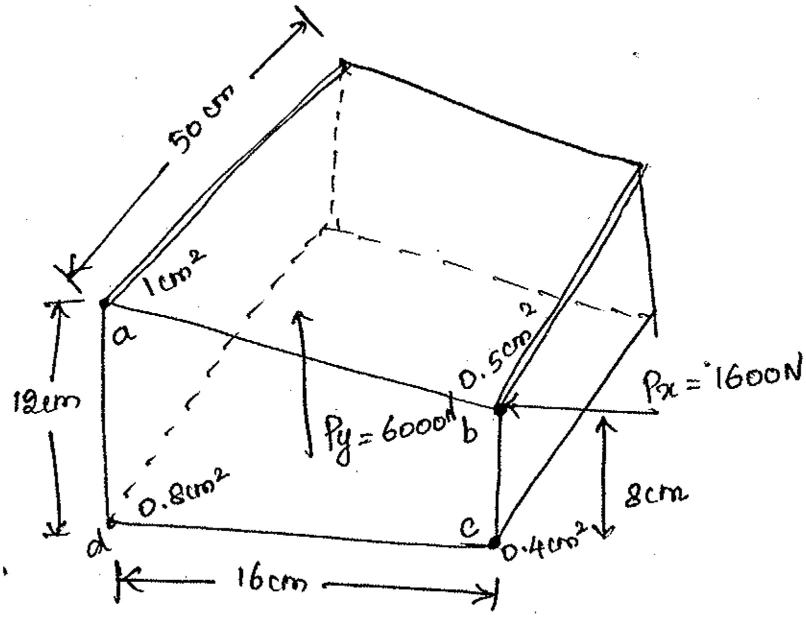
$q_{FG} = 4.318$ N/cm

$q_{GH} = 1.818$ N/cm

$q_{HA} = 0.568$ N/cm



Problem: 16 Fig shows a four flange Unsymmetrical single cell beam carrying two external loads as shown. find the shear flow distribution




$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{(0.5\times16)+(0.4\times16)}{1+0.8+0.4+0.5} = 5.333\text{ cm}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{(1\times12)+(0.5\times8)}{1+0.8+0.4+0.5} = 5.926\text{ cm}$$

$$I_{xx} = \Sigma Ay^2 = 1\times(12-5.926)^2 + 0.5\times(8-5.926)^2$$
$$+\,0.4\times(-5.926)^2 + 0.8(-5.926)^2$$
$$= 81.1852\text{ cm}^4$$

$$I_{yy} = \Sigma Ax^2 = 1\times(-5.333)^2 + 0.5\,(16-5.333)^2 + 0.4\times(16-5.33)^2$$
$$+\,0.8(-5.333)^2 = 153.6\text{ cm}^4.$$

$$I_{xy} = \Sigma Axy = 1\times -5.333\times(12-5.926) + (0.5)(16-5.333)$$
$$(8-5.926) + 0.4(16-5.333)\times -5.926 +$$
$$0.8\times -5.333 + -5.926$$
$$= -21.333\text{ cm}^4.$$

$V_y = 6000\,N$ and $V_x = -1600\,N$

$$q = \frac{(V_y I_{xy} - V_x I_x)\sum A_i x_i + (V_x I_{xy} - V_y I_y)\sum A_i y_i}{I_{xx} I_{yy} - I_{xy}^2}$$

$$= \frac{[6000 \times -21.333 - (-1600) \times 81.1852]\sum A_i x_i + [-1600 \times -21.33 - 6000 \times 153.6]\sum A_i y_i}{81.1852 \times 153.6 - 21.333^2}$$

$$\boxed{q = 0.157846853 \sum A_i x_i - 73.86361807 \sum A_i y_i}$$

Cut and open the section between a and b

$q_{ad}$ ; $A = 1\,cm^2$ ; $x_i = -5.333\,cm$ ; $y_i = (12 - 5.926)\,cm$

$q_{ad} = -449.4894\,N/cm$

$q_{dc}$ ; $A = 0.8\,cm^2$ ; $x_i = -5.333\,cm$ ; $y_i = -5.926\,cm$

$q_{dc} = -99.99\,N/cm$

$q_{cd}$ ; $A = 0.4\,cm^2$ ; $x_i = (16 - 5.333)\,cm$ ; $y = -5.926\,cm$

$q_{cd} = 75.7696\,N/cm$.

$q_{ba}$ ; $A = 0.5\,cm^2$ ; $x_i = (16 - 5.333)\,cm$ ; $y_i = (8 - 5.926)\,cm$

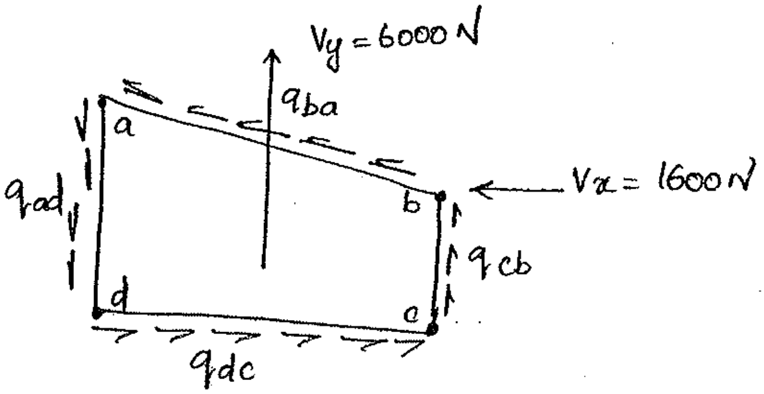
$q_{ba} = 0.0149\,N/cm$

Let $q_0$ be the indeterminate shear flow at the cut.

For uncut section

$q_{ad} = q_0 - 449.4894$ | $q_{cb} = q_0 + 75.7696$

$q_{dc} = q_0 - 99.99$ | $q_{ba} = q_0 + 0.0149$.




Taking moment about d;

$$\Sigma M_d \Rightarrow (q_0 + 0.0149)\, 2 \times \left(\frac{12 \times 16}{2}\right) + (q_0 + 75.7696)(8 \times 16)$$
$$= (6000 \times 8) + (1600 \times 8)$$

$$320 q_0 = 51098.6304$$

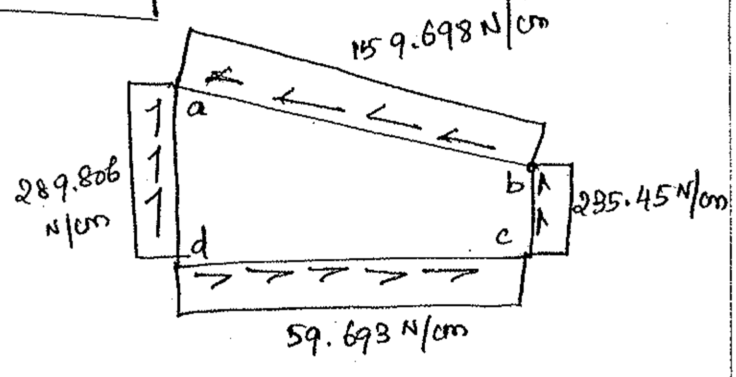
$$\boxed{q_0 = 159.68322\ N/cm}$$

$q_{ab} = -289.806\ N/cm$

$q_{dc} = 59.693\ N/cm$

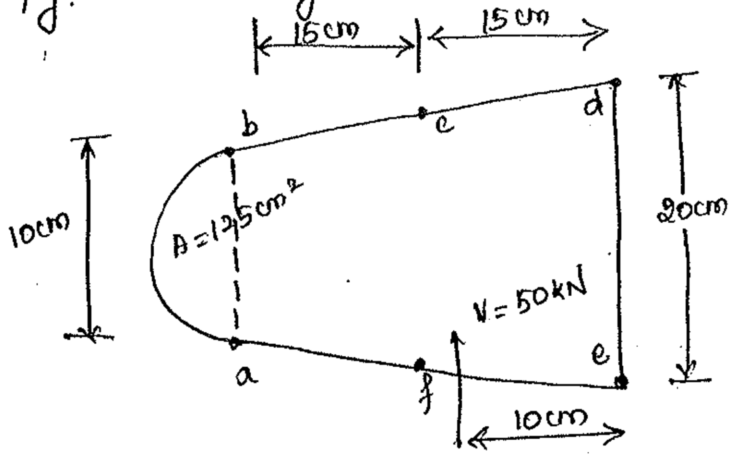
$q_{cb} = 235.4528\ N/cm$

$q_{ba} = 159.698\ N/cm$




Problem: 17

obtain the shear flow distribution for a closed section shown in fig. Each stringer area is 6.5 cm²



$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{(6.5 \times 15) \times 2 + (6.5 \times 30) 2}{6.5 \times 6} = 15\text{cm}$$

$$I_{xx} = \Sigma A y^2 = (6.5 \times 5^2) 2 + (6.5 \times 7.5^2) 2 + (6.5 \times 10^2) \times 2$$

$$= 2356.25 \text{ cm}^4$$

Cut & open the section just before "a"

$$q_{ab} = -\frac{V}{I} \Sigma A_i y_i = \frac{-50}{2356.25} \times 6.5 \times (-5) = 0.689655 \text{ KN/cm}$$

$$q_{bc} = 0.689655 - \frac{50}{2356.25} \times 6.5 \times 5 = 0$$

$$q_{cd} = 0 - \frac{50}{2356.26} \times 6.5 \times 7.5 = -1.0344827 \text{ KN/cm}$$

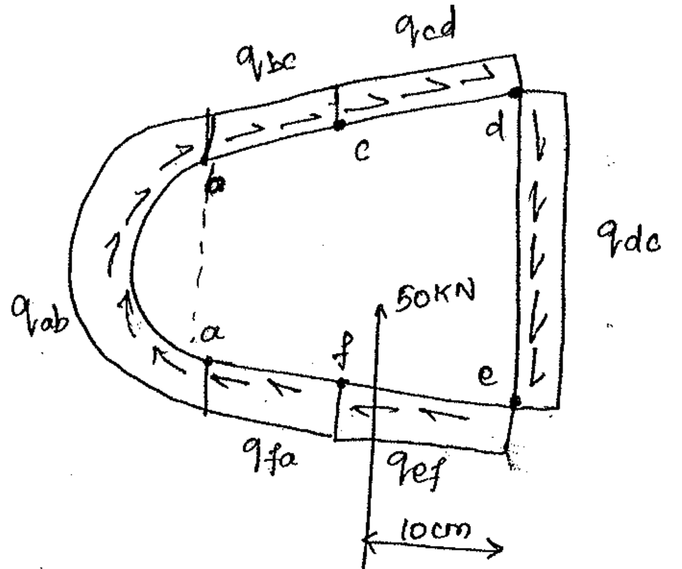
$$q_{de} = -1.0344 - \frac{50}{2356.25} \times 6.5 \times 10 = -2.413793 \text{ KN/cm}$$

$$q_{ef} = -2.413793 - \frac{50}{2356.25} \times 6.5 \times (-10) = -1.034482 \text{ KN/cm}$$

$$q_{fa} = -1.034482 - \frac{50}{2356.25} \times 6.5 (-7.5) = 0$$

For, uncut section

| | |
|---|---|
| $q_{ab} = q_0 + 0.689655$ | $q_{de} = q_0 - 2.413739$ |
| $q_{bc} = q_0$ | $q_{ef} = q_0 - 1.0344827$ |
| $q_{cd} = q_0 - 1.0344827$ | $q_{fa} = q_0$ |




Taking moment about f:

$$q_{ab} \times 2\left[125 \times \frac{10 \times 15}{2}\right] + q_{bc} \times 2\left[\frac{15 \times 15}{2}\right] + q_{cd} \times 2\left[\frac{15 \times 15}{2}\right]$$

$$+ q_{de} \times 2\left[\frac{20 \times 15}{2}\right] = -50 \times 5$$

$$(q_0 + 0.689655) \times 400 + 225 q_0 + (q_0 - 1.0344827)225$$

$$+ (q_0 - 2.4137931)300 = -250$$

$$1150\, q_0 = 431.034551$$

$$q_0 = 0.374812 \text{ KN/cm.}$$

$q_{ab} = 1.06446$ KN/cm

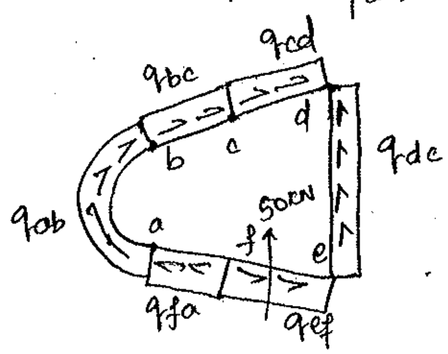
$q_{bc} = 0.374812$ KN/cm

$q_{cd} = -0.65967$ KN/cm

$q_{de} = -2.0389$ KN/cm

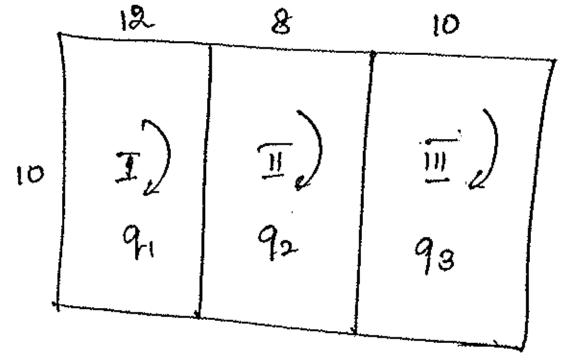
$q_{ef} = -0.65967$ KN/cm

$q_{fa} = 0.374812$ KN/cm



Problem : 18

Find the shear flow and twist per unit length for the section given below. Give $T = 20000$ N-cm. $G = 25$ GPa; thickness throughout the beam is 1mm;




data

$t = 1mm = 0.1cm$

$G = 25GPa = 25 \times 10^9 N/m^2$

$= \frac{25 \times 10^9}{10^4} = N/cm^2$

$G = 25 \times 10^5 N/cm^2$

$A_1 = 10 \times 12 = 120 cm^2$

$A_2 = 10 \times 8 = 80 cm^2$

$A_3 = 10 \times 10 = 100 cm^2$

$T = 2A_1 q_1 + 2A_2 q_2 + 2A_3 q_3$

$20000 = 240 q_1 + 160 q_2 + 200 q_3 \longrightarrow (1)$

For I cell

$\beta_1 = \frac{1}{2A_1 G t} \left[10q_1 + 12q_1 + 12q_1 + 10(q_1 - q_2)\right]$

$\beta_1 = 1.66 \times 10^{-8} \left[44 q_1 - 10q_2\right] \rightarrow (2)$

For II cell ; $\beta_2 = \frac{1}{2A_2 G t} \left[8q_2 + 10(q_2 - q_3) + 8q_2 + 10(q_2 - q_1)\right]$

$\beta_2 = 2.5 \times 10^{-8} (36q_2 - 10q_3 - 10q_1) \longrightarrow (3)$

For III cell;

$\beta_3 = \frac{1}{2A_3 G t} \left[10q_3 + 10q_3 + 10q_3 + 10(q_3 - q_2)\right]$

$\beta_3 = 2 \times 10^{8} \left[40q_3 - 10q_2\right] \longrightarrow (4)$

For compatibility condition

$$\beta_1 = \beta_2 \; ; \; \beta_2 = \beta_3 \; ; \; \beta_3 = \beta_1$$

$\beta_1 = \beta_2 \Rightarrow$

$$1.66\times10^{-8}(44q_1 - 10q_2) = 2.5\times10^{-8}(36q_2 - 10q_3 - 10q_1)$$

$$44q_1 - 10q_2 = 54.36q_2 - 15.1q_3 - 15.19q_1$$

$$59.1q_1 - 64.36q_2 + 15.1q_3 = 0 \longrightarrow (5)$$

$\beta_2 = \beta_3 \rightarrow$

$$2.5\times10^{-8}\left[36q_2 - 10q_3 - 10q_1\right] = 2\times10^{-8}(40q_3 - 10q_2)$$

$$45q_2 - 12.5q_3 - 12.5q_1 = 40q_3 - 10q_2$$

$$-12.5q_1 + 55q_2 - 52.5q_3 = 0 \longrightarrow (6)$$

Solving (1), (5) & (6)

$q_1 = 32.52$ N/cm

$q_2 = 37.19$ N/cm

$q_3 = 31.21$ N/cm.

Substituting these values in any of the above eqns.

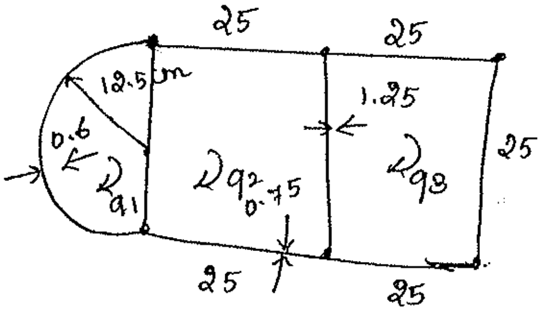
$$\beta = 1.66\times10^{-8}\left[44q_1 - 10q_2\right]$$

$$= 1.66\times10^{-8}\left[44\times32.52 - 10\times37.19\right]$$

$$\boxed{\beta = 17.5\times10^{-6} \text{ rad/m}}$$

Problem: 19.

Determine the internal shear flow for the given multi cell structure, when it is subjected to a torque of 100 kN-m. Thickness of vertical, horizontal and semi circular members are 1.25, 0.75 & 0.6 cm.




All dimensions are in cm.

$T = 100\ KN\text{-}m = 100\times10^3 N \times 10^{-2} cm$

$T = 100\times10^5\ N\text{-}cm$.

Soln

$A_1 = \frac{\pi}{2} r^2 = \frac{\pi}{8} \times 12.5^2 = 245.44\ cm^2$

$A_2 = 25\times25 = 625\ cm^2$ | $A_3 = 25\times25 = 625\ cm^2$

$T = 2A_1q_1 + 2A_2q_2 + 2A_3q_3$

$100\times10^5 = 490.87q_1 + 1250q_2 + 1250q_3 \rightarrow (1)$

I cell;
$$\beta_1 = \frac{1}{2A_1G}\left[\frac{\pi\times r}{0.6}q_1 + \frac{25(q_1-q_2)}{1.25}\right]$$

$$= \frac{1}{490.87G}\left[\frac{\pi\times12.5}{0.6}q_1 + \frac{25q_1}{1.25} - \frac{25q_2}{1.25}\right]$$

$$\beta_1 = \frac{1}{490.87G}\left[85.44q_1 - 20q_2\right] \rightarrow (2)$$

II cell;
$$\beta_2 = \frac{1}{2A_2G}\left[\frac{25q_2}{0.75} + \frac{25q_2}{0.75} + \frac{25(q_2-q_1)}{1.25} + \frac{25(q_2-q_3)}{1.25}\right]$$

$$\beta_2 = \frac{1}{1250G}\left[106.66q_2 - 20q_1 - 20q_3\right] \rightarrow (3)$$

III cell;
$$\beta_3 = \frac{1}{2A_3G}\left[\frac{25q_3}{0.75} + \frac{25q_3}{0.75} + \frac{25q_3}{1.25} + \frac{25(q_3-q_2)}{1.25}\right]$$

$$\beta_3 = \frac{1}{1250G}\left(120q_3 - 20q_2\right) \rightarrow (4)$$

For compatibility condition

$$\beta_1 = \beta_2 ; \quad \beta_2 = \beta_3 ; \quad \beta_3 = \beta_1$$

$\beta_1 = \beta_2 \Rightarrow$

$$\frac{1}{490.87G}\left[85.44q_1 - 20q_2\right] = \frac{1}{1250G}\left[106.66q_2 - 20q_1 - 20q_3\right]$$

$$85.44q_1 - 20q_2 = 41.92q_2 - 7.86q_1 - 7.86q_3$$

$$93.3q_1 - 61.92q_2 + 7.86q_3 = 0 \rightarrow (5)$$

$\beta_2 = \beta_3 \Rightarrow$

$$\frac{1}{1250G}\left[106.66q_2 - 20q_1 - 20q_3\right] = \frac{1}{1250G}\left(120q_3 - 20q_2\right)$$

$$-20q_1 + 126.66q_2 - 140q_3 = 0 \rightarrow (6)$$

$\beta_3 = \beta_1 \Rightarrow$

$$\frac{1}{1250G}\left(120q_3 - 20q_2\right) = \frac{1}{490.87G}\left(85.44q_1 - 20q_2\right)$$

$$120q_3 - 20q_2 = 217.53q_1 - 50.92q_2$$

$$217.53q_1 - 30.92 - 120q_3 = 0 \rightarrow (7)$$

Solving (1), (5) & (6)

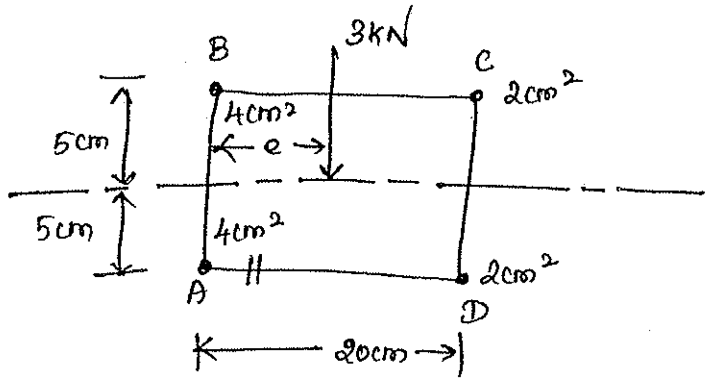
$q_1 = 2316.71$ N/cm

$q_2 = 3896.22$ N/cm

$q_3 = 3194.01$ N/cm

Problem: 20 Shear flow in Multicell under Bending

Calculate the shear flow and shear centre for the section given below




Soln:

| Points | Area | Y |
|---|---|---|
| A | 4 | -5 |
| B | 4 | 5 |
| C | 2 | 5 |
| D | 2 | -5 |

WKT ; $q = -\frac{V}{I} \sum A_i y_i$

$I = \sum A_i y_i^2$

$= 4\times5^2 + 4\times5^2 + 2\times5^2 + 2\times5^2$

$\boxed{I = 300 cm^4}$

$V = 3KN = 3000N.$

$$\left.\begin{aligned}
q_{AB} &= \frac{-V}{I}\sum A_i y_i = \frac{-3000}{300}(4\times-5) = 200\ N/cm \\
q_{BC} &= -\frac{3000}{300}(4\times5) + q_{AB} = 0. \\
q_{CD} &= -\frac{3000}{300}(2\times5) = -100 + q_{BC} = -100\ N/cm \\
q_{DA} &= -\frac{3000}{300}(2\times-5) + q_{CD} = 0
\end{aligned}\right\} \rightarrow (1)$$

(1) $\Rightarrow$ Statically determinate shear flow

$$\left.\begin{aligned}
q_{AB} &= q_0 + 200 \\
q_{BC} &= q_0 \\
q_{CD} &= q_0 - 100 \\
q_{DA} &= q_0
\end{aligned}\right\} \longrightarrow (2)$$

Equate $\beta = 0 \Rightarrow$

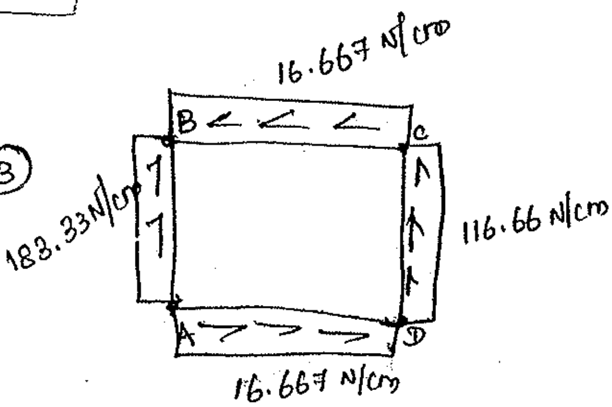
$$\beta = \frac{1}{2AGt}\left[(q_0+200)10 + (q_0 \times 20) + (q_0-100)10 + 20q_0\right] = 0$$

$$10q_0 + 2000 + 20q_0 + 10q_0 - 1000 + 20q_0 = 0$$

$$60q_0 = -1000$$

$$\boxed{q_0 = -16.667 \text{ N/cm}}$$

$$\left.\begin{aligned} q_{AB} &= 188.33 \text{ N/cm} \\ q_{BC} &= -16.667 \text{ N/cm} \\ q_{CD} &= -116.667 \text{ N/cm} \\ q_{DA} &= -16.667 \text{ N/cm} \end{aligned}\right\} \rightarrow (3)$$




To find shear centre:

$$\Sigma M_A = 0$$

$$3000e = -16.667 \times 20 \times 10 - 116.667 \times 20 \times 10$$

$$\boxed{e = -8.88 \text{ cm}}$$

$$\text{Torque} = -3000 \times -8.88$$

$$\boxed{T = 26.66 \times 10^3 \text{ N-cm}}$$

$$T = 2Aq$$

$$q_t = \frac{T}{2A}$$

$$q_t = \frac{26.66 \times 10^3}{2 \times (l \times b)} = \frac{26.66 \times 10^3}{2(20 \times 10)}$$

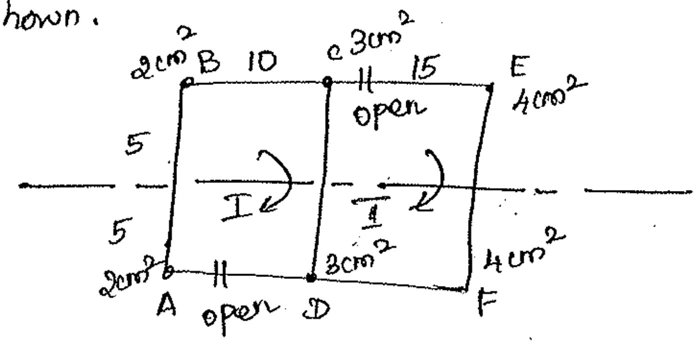
$$q_t = 66.667 \text{ N/cm.}$$

$q_{net}$ = Statically determinate shear flow + $q_0 + q_t \rightarrow (4)$

$q_{AB} = 250$ N/cm ; $q_{BC} = 50$ N/cm

$q_{CD} = -50$ N/cm ; $q_{DA} = 50$ N/cm

Problem 2) Find the shear flow and shear centre for the section shown.




| Point | Area | Y |
|---|---|---|
| A | 2 | -5 |
| B | 2 | 5 |
| C | 3 | 5 |
| D | 3 | -5 |
| E | 4 | 5 |
| F | 4 | 5 |

$$I_{xx} = \Sigma A_i y_i^2$$
$$= 2(2\times5^2) + 2(3\times5^2) + 2(4\times5^2)$$
$$= 450 cm^4.$$

$$q = \frac{-V}{I} A_i y_i$$

I cell

$$q_{AB} = \frac{-4500}{450}(2\times-5) = 100\ N/cm$$

$$q_{BC} = \frac{-4500}{450}(2\times5) + q_{AB} = 0$$

$$q_{CD} = \frac{-4500}{450}(3\times5) + q_{BC} = -150\ N/cm$$

$$q_{DA} = \frac{-4500}{450}(3\times-5) + q_{CD} = 0.$$

II cell

$$q_{EF} = \frac{-4500}{450}(4\times5) = -200\ N/cm$$

$$q_{FD} = \frac{-4500}{450}(4\times-5) + q_{EF} = 0$$

$$q_{DC} = \frac{-4500}{450}(3\times-5) + q_{FD} = 150\ N/cm$$

$$q_{CE} = \frac{-4500}{450}(3\times5) + q_{DC} = 0$$

$q_{01} \rightarrow$ Bending shear flow at opened section of cell I

$q_{02} \rightarrow$ Bending shear flow at opened section of cell II

Cell - I

$$\left.\begin{aligned} q_{AB} &= q_{01} + 100 \\ q_{BC} &= q_{02} \\ q_{CD} &= q_{01} - q_{02} - 150 \\ q_{DA} &= q_{10} \end{aligned}\right\} \rightarrow (1) \qquad \left.\begin{aligned} q_{EF} &= q_{02} - 200 \\ q_{FD} &= q_{02} \\ q_{DC} &= q_{02} - q_{01} + 150 \\ q_{CE} &= q_{20} \end{aligned}\right\} \rightarrow (2)$$

$$\beta_1 = \frac{1}{2GAt}\left[(q_{01} + 100) \times 10 + q_{01}(10) + (q_{01} - q_{02} - 150)\,10 + q_{01} \times 10)\right] = 0$$

$$10q_{01} + 1000 + 10q_{01} + 10q_{01} - 10q_{02} - 1500 + 10q_{01} = 0$$

$$40q_{01} - 10q_{02} = 500 \rightarrow (3)$$

$$\beta_2 = \frac{1}{2AGt}\left[(q_{02} - 200)10 + q_{02} \times 15 + (q_{02} - q_{01} + 150)\,10 + (q_{02} \times 15)\right] = 0.$$

$$10q_{02} - 2000 + 15q_{02} + 10q_{02} - 10q_{01} + 1500 + 15q_{02} = 0$$

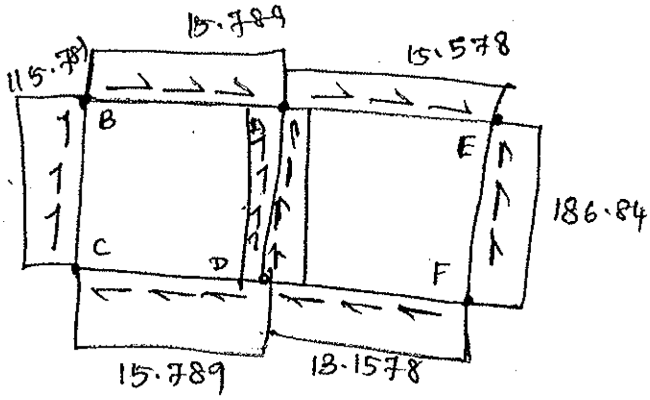
$$15q_{02} - 10q_{01} = 500 \rightarrow (4)$$

Solving (3) & (4)

$$\boxed{\begin{aligned} q_{01} &= 15.789 \text{ N/cm} \\ q_{02} &= 13.1578 \text{ N/cm} \end{aligned}}$$

Add these values to (1) & (2)

$$\left.\begin{aligned} q_{AB} &= 115.789 \text{ N/cm} \\ q_{BC} &= 15.789 \text{ N/cm} \\ q_{CD} &= -147.368 \text{ N/cm} \\ q_{DA} &= 15.789 \text{ N/cm} \end{aligned}\right\} \rightarrow (5) \qquad \left.\begin{aligned} q_{EF} &= -186.84 \text{ N/cm} \\ q_{FD} &= 13.1578 \text{ N/cm} \\ q_{DC} &= 147.3688 \text{ N/cm} \\ q_{CE} &= 15.1578 \text{ N/cm} \end{aligned}\right\} \rightarrow (6)$$




To find Shear centre : $\Sigma M_A = 0$

$$4500\,e = (15.789 \times 10 \times 10) - (147.368 \times 10) \times 10 + (15.578 \times 15 \times 10) - (186.84 \times 10 \times 25) + (147.3688) \times 10 \times 10) = -9.574 \text{ cm}$$

To find $q_t$ :

$T = -4500 \times e = 43065$

$T = 43065$ N-cm

$T = 2Aq$ ; $q_t = \frac{T}{2A} = \frac{43065}{2 \times 25 \times 10}$ $(A = l \times b)$

$q_t = 86.13$ N/cm.

$q_{net} = q_b + q_t$

③ ⇒ $q_{AB} = 201.91$ N/cm

$q_{BC} = 101.919$ N/cm

$q_{CD} = -61.238$ N/cm

$q_{DA} = 101.919$ N/cm

④ ⇒

$q_{EF} = -100.71$ N/cm

$q_{FD} = 99.2878$ N/cm

$q_{DC} = 238.4988$ N/cm

$q_{CE} = 99.28$ N/cm